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VERTICAL AND HORIZONTAL SEQUENCES FOR ORGANIZING MATHEMATICS CONTENT IN ELEMENTARY GRADES FOUR, FIVE, AND SIX WERE IDENTIFIED USING AN EMPIRICAL BASE. VERTICAL SEQUENCES FOR THIS PROJECT PERTAINED TO SUBTOPICS WITHIN SEVERAL MATHEMATICS TOPICS USING THE LEVEL OF SUBTOPIC COMPLEXITY FOR THE CRITERION. THE DEGREE OF RELATIONSHIP BETWEEN ADJACENT TOPICS WAS THE CRITERION FOR HORIZONTAL SEQUENCES WITH BOTH THE ORTHOGONAL AND NON ORTHOGONAL APPROACHES. A TEST DESIGNED TO ASSESS ACHIEVEMENT OF MATHEMATICS INSTRUCTIONAL OBJECTIVES WAS ADMINISTERED TO APPROXIMATELY 760 SUBJECTS, DIVIDED ALMOST EVENLY AMONG THE 3 GRADE LEVELS USED. SIMPLEX AND 'SMALLEST SPACE' ANALYSES WERE THE BASIC MEANS USED TO IDENTIFY THE VERITCAL AND HORIZONTAL SEQUENCES, RESPECTIVELY. THE VERTICAL HIERARCHIAL ORDERS AMONG SUBTOPICS WERE SHOWN FROM LEAST TO MOST COMPLEX FOR THE FOLLOWING MAJOR TOPICS--(1) NUMBERS, OPERATIONS, AND ASSUMPTIONS, (2) GEOMETRY (FIFTH AND SIXTH GRADES ONLY), (3) RELATIONS, (4) NUMERATION, (5) MEASUREMENT, AND(6) FRACTIONS (SIXTH GRADE ONLY). THE TOPICS OF GEOMETRY AND OPERATIONS WITH WHOLE NUMBERS WERE SET OFF FROM THE OTHERS AND FROM EACH OTHER IN THE HORIZONTAL SEQUENCES. THE BEST ORTHOGONAL TOPIC SEQUENCE AND NON ORTHOGONAL TOPIC SEQUENCE WERE PROVIDED. THE BASIS FOR HORIZONTAL RELATIONSHIPS IDENTIFIED WAS THE DEGREE OF COMMONALITY REPRESENTED BY PROBLEM-SOLVING ELEMENTS IN EACH TOPIC. (JH)

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ORGANIZATION OF MATHEMATICS IN GRADES 4, 5, AND 6

Cooperative Research Project No. 2531

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College of Education, University of Georgia
and the
Clarke County (Georgia) School District

Athens, Georgia
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PREFACE

The investigation reported herein was conducted pursuant to contract for Cooperative Research Project Number 2531. The work was supported through the Cooperative Research Program of the Office of Education, U. S. Department of Health, Education, and Welfare.

Many people participated in conducting and reporting this project. It is a pleasure to acknowledge and express appreciation to those named below. Their cooperation and assistance contributed greatly to the completion of the work.

Mr. Sam W. Wood, Superintendent of the Clarke County (Georgia) School District, gave clearance for the project to be conducted in the School District and allocated work-load time for Mr. Ellis to devote to the project. Work-load time for Dr. Blake, Dr. Findley, and Dr. Westbrook to devote to the project, equipment, and work space were allocated by the following: Dr. Joseph A. Williams, Dean of the College of Education at the University of Georgia; Dr. Stanley H. Ainsworth, Chairman of the Program for Exceptional Children; and Dr. Robert A. McRorie, Director of the Office of General Research.

Several consultants were helpful in solving problems related to sampling mathematics achievement, developing instruments, and using the scaling procedures. These consultants included Dr. Louis Guttman at the Hebrew University and the Israel Institute of Applied Social Research, Dr. Henry F. Kaiser at the University of Wisconsin, Dr. James C. Lingoes at the University of Michigan, and the following University of Georgia professors: Dr. Harry E. Anderson, Dr. James L. Carmon, Mr. James C. Fortson, Dr. James B. Kenney, Dr. Albert J. Kingston, Dr. Leonard E. Pikaart, and Dr. W. Owen Scott. In addition, Dr. Lingoes and personnel at the University of Michigan Computer Center performed the computer operations for the Smallest Space Analysis I and the Smallest Space Analysis III.

Many public school pupils participated in the data collection phase of the project. The following are cooperating public school personnel who facilitated the work of the project with pupils under their jurisdiction: Dr. J. C. Mullis, Director of Testing and Guidance, and principals of several elementary schools -- Miss Allea Betts,

Miss Dorothy Firor, Mr. Robert Garrard, Miss Grace Hancock, Mr. Donald Hight, Mrs. Rosa Tarpley, Miss Helen Treanor, and Miss Annie Wallace. Appendix A contains the names of the schools from which pupils were selected and the names of the teachers who administered the tests to the subjects.

Mrs. Norma Galbreath coordinated and supervised activities involved in the preparation of the research materials. Mr. Robert M. Brown did the background research and performed the reliability analyses and the simplex analyses. He was assisted by Mr. L. Wayne Swindel. Personnel who scored research instruments and encoded data were Miss Joan Bond, Mr. Michael Bradley, and Miss Sharron Riser. Mr. Bradley, Miss Malese Anderson, Mrs. Margaret Jones, Mrs. Lynn Monroe, and Mrs. Dana Stonesifer assisted with the preparation of the final report.

The Educational Testing Service and the California Test Bureau granted permission for the use of selected test items in the research instruments.

While those who are named in this prefatory statement made valuable contributions to the project, the project directors alone bear the entire and final responsibility for any errors and inadequacies in planning, execution, interpretation, and reporting.

Kathryn Blake
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CONTENTS

Page

PREFACE iii

LIST OF TABLES vii

LIST OF FIGURES xiii

Chapter

I. PROBLEM 1

Purpose and Objectives 1
Educational Significance 1
Related Literature 2

II. PROCEDURE 5

Research Design 5
Definitions of Terms 5
Subjects 6
The Mathematics Achievement Test 10

Description of the Test 10
Reliability of the Mathematics Topic and Subtopic Scores 13
General Procedures 13

III. THE VERTICAL SEQUENCES 19

Research Objective and Procedures for Analyses 19
Results 20

Number, Operations, and Assumptions 20
Geometry 29
Relations 35
Numeration 43
Measurement 51
Fractions 59

Summary of Conclusions 63

IV. THE HORIZONTAL SEQUENCES 65

Research Objective and Procedures for Analyses 65
Smallest Space Analysis of Topics 65
Smallest Space Analysis of Subtopics 71
Implications of Analyses 79
Summary of Conclusions 80

V. SUMMARY 81

Problem 81

Purpose and Objectives 81

Chapter Page

Educational Significance 81
Related Literature 81

Procedure 82

Research Design 82
Subjects 82
The Mathematics Achievement Test 82
General Procedures 83

The Vertical Sequences 83

The Horizontal Sequences 84

Conclusions and Limitations 84

Conclusions Relevant to the Vertical Sequences 84
Conclusions Relevant to the Horizontal Sequences 85
Limitations 86

REFERENCES 89

APPENDICES A1

A. COOPERATING SCHOOL PERSONNEL A3

B. INSTRUCTIONAL OBJECTIVES AND SELECTED TEST ITEMS A7

C. SUPPLEMENTARY MATHEMATICS TEST A37

D. SOURCE AND IDENTIFICATION OF MATHEMATICS TEST ITEMS A53

E. DESCRIPTIVE AND INFERENTIAL STATISTICS: MEANS AND VARIANCES FOR THE SUBJECTS' PERFORMANCE ON THE MATHEMATICS TEST A59

F. PRINCIPAL AXES COORDINATES FOR TOPICS A71

LIST OF TABLES

Table | Page

1. Number of Classes Selected from Total Number of Available Classes Categorized by School and Grade Level 9

2. Composition of Mathematics Test by Topic, Subtopic, and Number of Items 11

3. Reliability Coefficients for the Mathematics Topic and Subtopic Scores 14

4. Grade 4 -- Number, Operations, and Assumptions: Matrix of Observed r_{jk}, Subtopics in the Expected Order 23

5. Grade 4 -- Number, Operations, and Assumptions: Matrix of Observed R_{jk}, and the A_j, a_j, and q^2 values 23

6. Grade 4 -- Number, Operations, and Assumptions: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 24

7. Grade 4 -- Number, Operations, and Assumptions: Matrix of (a_j/a_k), the Reproduced r_{jk} 24

8. Grade 5 -- Number, Operations, and Assumptions: Matrix of Observed r_{jk}, Subtopics in the Expected Order 25

9. Grade 5 -- Number, Operations, and Assumptions: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 25

10. Grade 5 -- Number, Operations, and Assumptions: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 26

11. Grade 5 -- Number, Operations, and Assumptions: Matrix of (a_j/a_k), the Reproduced r_{jk} 26

12. Grade 6 -- Number, Operations, and Assumptions: Matrix of Observed r_{jk}, Subtopics in the Expected Order 27

13. Grade 6 -- Number, Operations, and Assumptions: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 27

14. Grade 6 -- Number, Operations, and Assumptions: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 28

15. Grade 6 -- Number, Operations, and Assumptions: Matrix of (a_j/a_k), the Reproduced r_{jk} 28

16. Grade 5 -- Geometry: Matrix of Observed r_{jk}, Subtopics in the Expected Order 31

Table Page

17. Grade 5 -- Geometry: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 31

18. Grade 5 -- Geometry: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 32

19. Grade 5 -- Geometry: Matrix of (a_j/a_k), the Reproduced r_{jk} 32

20. Grade 6 -- Geometry: Matrix of Observed r_{jk}, Subtopics in the Expected Order 33

21. Grade 6 -- Geometry: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 33

22. Grade 6 -- Geometry: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 34

23. Grade 6 -- Geometry: Matrix of (a_j/a_k), the Reproduced r_{jk} 34

24. Grade 4 -- Relations: Matrix of Observed r_{jk}, Subtopics in the Expected Order 37

25. Grade 4 -- Relations: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 37

26. Grade 4 -- Relations: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 38

27. Grade 4 -- Relations: Matrix of (a_j/a_k), the Reproduced r_{jk} 38

28. Grade 5 -- Relations: Matrix of Observed r_{jk}, Subtopics in the Expected Order 39

29. Grade 5 -- Relations: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 39

30. Grade 5 -- Relations: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 40

31. Grade 5 -- Relations: Matrix of (a_j/a_k), the Reproduced r_{jk} 40

32. Grade 6 -- Relations: Matrix of Observed r_{jk}, Subtopics in the Expected Order 41

33. Grade 6 -- Relations: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 41

Table Page

34. Grade 6 -- Relations: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 42

35. Grade 6 -- Relations: Matrix of (a_j/a_k), the Reproduced r_{jk} 42

36. Grade 4 -- Numeration: Matrix of Observed r_{jk}, Subtopics in the Expected Order 45

37. Grade 4 -- Numeration: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 45

38. Grade 4 -- Numeration: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 46

39. Grade 4 -- Numeration: Matrix of (a_j/a_k), the Reproduced r_{jk} 46

40. Grade 5 -- Numeration: Matrix of Observed r_{jk}, Subtopics in the Expected Order 47

41. Grade 5 -- Numeration: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 47

42. Grade 5 -- Numeration: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 48

43. Grade 5 -- Numeration: Matrix of (a_j/a_k), the Reproduced r_{jk} 48

44. Grade 6 -- Numeration: Matrix of Observed r_{jk}, Subtopics in the Expected Order 49

45. Grade 6 -- Numeration: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 49

46. Grade 6 -- Numeration: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 50

47. Grade 6 -- Numeration: Matrix of (a_j/a_k), the Reproduced r_{jk} 50

48. Grade 4 -- Measurement: Matrix of Observed r_{jk}, Subtopics in the Expected Order 53

49. Grade 4 -- Measurement: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 53

50. Grade 4 -- Measurement: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 54

Table Page

51. Grade 4 -- Measurement: Matrix of (a_j/a_k), the Reproduced r_{jk} 54

52. Grade 5 -- Measurement: Matrix of Observed r_{jk}, Subtopics in the Expected Order 55

53. Grade 5 -- Measurement: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 55

54. Grade 5 -- Measurement: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 56

55. Grade 5 -- Measurement: Matrix of (a_j/a_k), the Reproduced r_{jk} 56

56. Grade 6 -- Measurement: Matrix of Observed r_{jk}, Subtopics in the Expected Order 57

57. Grade 6 -- Measurement: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 57

58. Grade 6 -- Measurement: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 58

59. Grade 6 -- Measurement: Matrix of (a_j/a_k), the Reproduced r_{jk} 58

60. Grade 6 -- Fractions: Matrix of Observed r_{jk}, Subtopics in the Expected Order 61

61. Grade 6 -- Fractions: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 values 61

62. Grade 6 -- Fractions: Matrix of $(A_j - A_k)$, the Reproduced R_{jk} 62

63. Grade 6 -- Fractions: Matrix of (a_j/a_k), the Reproduced r_{jk} 62

64. Guttman-Lingoes Smallest Space Coordinates for Grade 4 .. 68

65. Guttman-Lingoes Smallest Space Coordinates for Grade 5 .. 68

66. Guttman-Lingoes Smallest Space Coordinates for Grade 6 .. 68

67. Guttman-Lingoes Smallest Space Coordinates for Six Topics in Grade 6 68

68. Smallest Space Analysis III for Subtopics in Grade 4 72

69. Smallest Space Analysis III for Subtopics in Grade 5 73

Table Page

70. Smallest Space Analysis III for 20 Subtopics in Grade 6 .. 74

71. Smallest Space Analysis III for 25 Subtopics in Grade 6 .. 75

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LIST OF FIGURES

Figure Page

1. Smallest Space Coordinates for Grade 4 69
2. Smallest Space Coordinates for Grade 5 69
3. Smallest Space Coordinates for Grade 6 70
4. Smallest Space Coordinates for Six Topics in Grade 6 70

CHAPTER I

PROBLEM

Purpose and Objectives

This study's purpose was to use procedures based on Guttman's (1954, 1965) radex theory in identifying,empirically, vertical and horizontal sequences for organizing mathematics content at grade levels 4, 5, and 6. The specific objectives were as follows:

1. to identify, at grade levels 4, 5, and 6, vertical sequences among subtopics within several mathematics topics on the basis of the criterion, level of complexity of the subtopics;
2. to identify, at grade levels 4, 5, and 6, orthogonal and non-orthogonal horizontal sequences among several mathematics topics on the basis of the criterion, degree of relationship between adjacent topics.

Educational Significance

Information yielded by this study and by similar studies could be useful in establishing instructional sequences for teaching mathematics. That is, more effective instructional sequences can be established when the structure of the interrelations within the material to be taught is considered jointly with such planning factors as the characteristics of the pupils and the practical exigencies of the situation.

Manning (1960, p. 116) succinctly portrayed how this planning process can be strengthened by identifying sequential arrangements among mathematics activities on the empirical basis of the achievement of pupils who have been taught in a given mathematics instructional program. In his introductory remarks about a prototype design for this research problem, he made these comments.

> The organization of topics within a course or the determination of the optimum sequence of courses in any area of learning is recognized as a fundamental educational problem. When faced with an apparently heterogeneous array of topics, the teacher must impose an order or arrangement which he feels will benefit the learner. Sometimes the order is dictated by the levels of complexity of the various topics; that is, the student must have mastered topic A before he can go on to topic B and so forth. In other instances the topics are related, but on an equivalent level of complexity, so that either A or B may be studied earlier.

> What is apparent in most instances is that the order or sequence of learning is largely determined on the basis of professional judgment. Although professional judgment is not inherently unsatisfactory, it would be preferable to structure a curriculum on the basis of the behavior of the learners themselves.

Related Literature

The present study's purpose essentially involved using procedures based on Guttman's radex theory (1954, 1965) to examine the structure of the interrelations among aspects of mathematics content taught at the upper elementary school grade levels. Accomplishing the study's purpose necessitated that the investigators obtain scores on tests of attainment of selected instructional objectives for a given mathematics instructional program for grade levels 4, 5, and 6. The present investigators located no other projects which were specifically designed to use Guttman's or other scaling procedures as adjuncts in establishing sequences among activities in contemporary mathematics instructional programs for elementary school pupils.

Guttman (1954) and Kaiser (1962) presented material which can be used as evidence relevant to interrelations among several mathematics activities. As illustrative materials for their expositions on the simplex analysis, they used mathematics data which they culled from the larger body of data Thurstone (1938) reported in his study of the primary mental abilities. Specifically, they demonstrated that the simplex model fits a set of children's numerical performance tests used by Thurstone. Among these tests, the hierarchical order from simple to complex was addition, subtraction, multiplication, division, arithmetical reasoning, and numerical judgment.

Material more tangentially related to the present project includes the work reported by Gabriel (1954), Guttman (1954, 1957, 1965), Humphreys (1960), and Lingoes (1965a). They showed that procedures related to Guttman's radex theory yielded satisfactory descriptions of order relations among various types of behavioral data. Gabriel used a simplex analysis to ascertain the structure of the Progressive Matrices Test. Guttman applied simplex analyses, circumplex analyses, and smallest space analyses to interrelations among sets of cognitive and personality variables, respectively. In these analyses, Guttman sampled intercorrelation matrices reported by various authors, e.g., Thurstone (1938, 1940, 1941) and Coombs (1951). Humphreys employed the simplex model in his consideration of the intercorrelations between repeated measures reported by various authors: namely, trials during the learning of two motor skills -- complex coordination (stick and rudder coordination task) and discrimination reaction time(Fleishman and Hempel, 1954; 1955); growth data for height and weight (Anderson, 1939); and periodic tests during shorthand training (Lewin, unpublished). Lingoes applied the smallest space analysis to data reflecting cognitive characteristics reported by Guilford and his associates (1952).

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CHAPTER II

PROCEDURE

Research Design

The plan for collecting data pertinent to the project's purpose involved administering tests designed to assess pupils' achievement of instructional objectives for a mathematics instructional program at grade levels 4, 5, and 6. Procedures used to carry out this plan are described more specifically below in terms of the following topics: definitions of terms, subjects, the mathematics achievement test, and general procedures.

Definitions of Terms

For the purposes of this study, the following definitions of terms[1] were employed.

1. Radex theory. A theory for investigating the order relationships among variables. The method seeks to determine from a pattern of intercorrelations the underlying order structure of the constituent variables (Guttman, 1954; 1965).
    a. Radex. A set of variables whose intercorrelations conform to the general order pattern prescribed by the theory. Two distinct notions are involved in a radex. One is that of a difference in kind between variables, and the other is that of a difference in degree (Guttman, 1954; 1965).
    b. Simplex. A set of variables which differ among themselves, largely in the degree of their complexity. A simplex possesses a simple order of complexity. The variables can be arranged in simple rank order from least complex to most complex (Guttman, 1954; 1965).
    c. Circumplex. A set of variables which differ among themselves in the kind of ability they define. The law of order here is not from "least" to "most" in any sense; it is an order which has no beginning and no end, namely, a circular order (Guttman, 1954; 1965).
2. Sequence. An order in which mathematics content is arranged for presentation to the pupils.

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[1] In the definitions of radex, simplex, and circumplex, the italics and quotation marks are Guttman's; in the definitions of the non-orthogonal and orthogonal sequences, the italics are the present investigators'.

a. Vertical sequence. An arrangement of the subtopics within any given topic in terms of the criterion, level of complexity of subtopics; level of complexity refers to the extent to which a given subtopic includes aspects of other subtopics.

b. Horizontal sequence. Arrangement of different topics in terms of the criterion, degree of relationship between adjacent topics.

    1) Orthogonal sequence. A horizontal sequence in which the different topics are arranged so that adjacent topics are the least closely related; *i.e.*, adjacent topics are relatively independent.

    2) Non-orthogonal sequence. A horizontal sequence in which the different topics are arranged so that adjacent topics are the most closely related; *i.e.*, adjacent topics are relatively non-independent.

3. Mathematics content[1]. The domain or universe of mathematics taught the pupils at any one grade level.

    a. Mathematics topics. Areas of mathematics content, for example: number, operations, assumptions.

    b. Mathematics subtopics. Specific subdivisions of content within the mathematics topics. For example, the topic--number, operations, assumptions--includes such subtopics as addition and multiplication.

    c. Mathematics instructional objectives. Precise specifications of behavior, relevant to the subtopics, to be attained through the instructional program, for example: understanding the concept of rational numbers, facility in adding two-place whole numbers.

Subjects

The subjects were 244 fourth grade pupils, 261 fifth grade pupils, and 260 sixth grade pupils. These pupils were selected from the total number of pupils enrolled at grade levels 4, 5, and 6 in the Clarke County (Georgia) School District during the 1963-64 school year. The number and sizes of the classes sampled and steps involved in the sampling process are specified below.

The total enrollment at each grade level consisted of approximately 500 pupils. These pupils attended eight elementary schools; each elementary school contained either two or three classes at each grade level. Class size ranged from 25 to 40 pupils.

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[1] These definitions are based on materials presented in Mathematics for Georgia Schools (Georgia Curriculum Guide Committee, 1962).

The sampling process consisted of two steps: (1) selecting participating classes; (2) selecting participating pupils within these classes. These two steps are described more fully below.

The first step in the sampling process was to select classes of pupils at each grade level. The selection of classes was based primarily on two criteria. The first criterion was that the classroom teacher had taught in the elementary schools in Clarke County School District for at least the previous three years. The second criterion was that the classroom teacher had been an active participant in the School District's in-service mathematics education program.

These two class selection criteria were used to insure that teachers of the prospective subjects had had training and experience in teaching mathematics as presented in Volumes 1, 2, and 3 of the state curriculum guide, Mathematics for Georgia Schools[1] (Georgia Curriculum Guide Committee, 1962). The Clarke County School District has been in the process of revising its mathematics program for the past few years, using the state curriculum guide as a framework to improve mathematics instruction in the local schools. The revision process has included various activities[1] designed to increase the effectiveness with which teachers teach mathematics.

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[1] This curriculum guide was developed, tried out, and revised over a period of several years. It was designed to serve as a framework for teaching mathematics in Georgia Schools. Members of the Georgia Mathematics Curriculum Guide Committee were personnel from throughout the state including the following: from the Georgia State Department of Education and the universities--mathematicians and mathematics educators; from the public school systems--instructional supervisors, principals, secondary school mathematics teachers, and elementary school teachers who had special training and competence in mathematics education. Consultants to the Committee were a number of mathematicians and mathematics educators from universities throughout the nation and from federal and private agencies.

Twenty-six classes were identified by applying the class selection criteria related to length of the teacher's service and to the teacher's participation in mathematics education activities. There were nine classes on the fourth grade level, eight classes on the fifth grade level, and nine classes on the sixth grade level. Table 1 contains data reflecting the distribution of these classes by schools. Appendix A consists of a list of school personnel who participated in the data collection procedures.

The second step in the sampling process was to identify pupils in the selected classes who were eligible to participate as subjects. Eligible to be subjects were pupils in the selected classes who were taught in the mathematics instructional program during the 1963-1964 school year and who satisfied the following additional selection criteria.

1. The pupil had no gross physical or sensory impairments which might interfere with responses to the mathematics instructional program or to the instruments used in the study.

2. The pupil had no major emotional or motivational difficulties which might interfere with responses to the mathematics instructional program or to the instruments used in the study.

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[1] Examples of activities included in this revision process were the following.

a. In-service teacher education programs were instituted; these included intensive work during the school year and during pre-school and post-school planning periods. Serving these in-service programs as consultants were mathematicians and mathematics educators from throughout the nation, the Georgia State Department of Education, and the University of Georgia.

b. Mathematics courses for teachers were taught by University of Georgia professors. The expenses incurred by teachers who participated in these courses were borne by the Clarke County Board of Education.

c. Closed circuit T.V. programs for teachers were presented; in these T.V. programs, master teachers in the Clarke County School District demonstrated methods for teaching modern mathematics.

Table 1

Number of Classes Selected From Total Number of Available Classes Categorized by School and Grade Level

	4th Grade		5th Grade		6th Grade	
School	Total Number of Classes	Number of Classes Selected	Total Number of Classes	Number of Classes Selected	Total Number of Classes	Number of Classes Selected
1	4	2	3	2	3	2
2	2	1	2	1	2	1
3	3	2	3	2	2	2
4	1	1	1	0	1	1
5	2	1	2	1	2	1
6	1	0	1	1	1	1
7	2	1	2	1	2	1
8	1	1	1	0	1	0
Total	16	9	15	8	14	9

Contacts were made with principals and teachers in order to locate pupils who did not meet the subject selection criteria. Pupils in the selected classes who did not meet the subject selection criteria were identified and designated for exclusion from the study. The tests were administered to the excluded pupils at the same time they were administered to the other pupils in the selected classes. After the tests were collected, the excluded pupils' tests were removed. The excluded pupils' scores were not tabulated nor included in the data analyses. The numbers of pupils who qualified as subjects were as follows: fourth grade -- 244, fifth grade -- 261, sixth grade -- 260.

The Mathematics Achievement Test

The mathematics achievement data were interval scores on a test containing items selected to reflect the extent to which the pupils attained instructional objectives for mathematics topics and subtopics which they were taught specifically. Details about the nature of the test are presented below.

Description of the Test

Generally, the selection of items for the mathematics test was limited to materials sampling the instructional objectives of the mathematics curriculum for the fourth, fifth, and sixth grades. Specifically, the test was designed to measure achievement of selected instructional objectives for subtopics within six topics: number, operations, and assumptions; geometry; relations; numeration; measurement; and fractions. Each subtopic was represented by four test items The composition of the test is portrayed in Table 2. This table contains the names of the topics and subtopics and the number of test items relevant to each.

The framework of the mathematics curriculum to which the subjects were exposed was the Georgia mathematics curriculum guide, Mathematics for Georgia Schools (Georgia Curriculum Guide Committee, 1962). This guide was considered to be a task analysis of the curriculum used in the Clarke County School District. The guide consists of topics, subtopics, instructional objectives, and specific teaching activities. Developing the mathematics test involved the following steps.

1. Reviewing in detail the curriculum guide, Mathematics for Georgia Schools.

Table 2

Composition of Mathematics Test by Topic, Subtopic, and Number of Items

Topic	Subtopic	Number of Items
Number, Operations, Assumptions	Addition	4
	Subtraction	4
	Multiplication	4
	Division	4
	Laws and Generalizations	4
		20 Total
Geometry	Definitions of geometric terms	4
	Recognition of geometric figures	4
	Measurement of geometric figures	4
		12 Total
Relations	Equality	4
	Order relations	4
	Maps, graphs, charts	4
	Functions	4
		16 Total
Numeration	Sets	4
	Other number bases	4
	Reading, writing, using numbers	4
	Roman numerals	4
		16 Total
Measurement	Concepts of Measurement	4
	Conversion of units	4
	Operations without conversion	4
	Operations with conversion	4
		16 Total
Fractions	Concepts of fractions	4
	Addition of fractions	4
	Subtraction of fractions	4
	Multiplication of fractions	4
	Division of fractions	4
		20 Total

2. Selecting six topics based on materials presented in the curriculum guide. Designating several subtopics relevant to each topic. For these topics and subtopics, further selecting instructional objectives from the total number of instructional objectives presented in the curriculum guide.
3. Reviewing appropriate standardized mathematics tests.
4. Choosing test materials to reflect the instructional objectives selected for study. For instructional objectives which were not represented by material in standardized tests, constructing test items.
5. Presenting the selected instructional objectives and the test materials to judges[1] for evaluation of the items' appropriateness, relevance, *etc.*
6. Making appropriate changes in the test materials on the basis of recommendations by the judges.

Appendix B consists of the following materials germane to these several steps: the instructional objectives selected for study, designations of the grade levels at which these instructional objectives apply, and the test items chosen to reflect the selected instructional objectives. These materials are organized by the topics and subtopics chosen for study.

Specifically, the mathematics test was composed of 100 items and required 110 minutes to administer. Forty of the 100 items consisted of selected items in Parts II and IV of the *School and College Ability Tests, Form 5A* (Educational Testing Service, 1957). Subjects were administered the published form of this test which contains a total of 50 items[2] in Parts II and IV. The remaining 60 test items needed were obtained from the following sources: *Sequential Tests of Educational Progress, Forms 4A, 4B* (Educational Testing Service, 1957), *Contemporary Mathematics Tests: Upper and Lower Elementary Levels, Form A* (California Test Bureau, 1963), and 18 items constructed especially for the present study. These 60 items were reproduced in mimeographed form and administered as a supplementary test. This supplementary test is in Appendix C. The source and identification of each of the 100 items selected for inclusion are presented in Appendix D.

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[1] These judges respectively were specialists in mathematics, tests and measurements, elementary education, and educational psychology

[2] Only 40 of the 50 items were included in the data analyses. Ten items were not relevant to the instructional objectives chosen for study.

Reliability of the Mathematics Topic and Subtopic Scores

The reliability coefficients for the mathematics topic and subtopic scores were calculated by the split-half method and corrected by the Spearman-Brown Prophecy Formula. The data were the scores of pupils at each grade level. The obtained reliability coefficients are presented in Table 3. Inspection of this table reveals that, in general, the reliabilities of the topic scores are sufficiently high to be considered satisfactory for research purposes with groups of subjects (Thorndike, 1951). However, it should be noted that the reliabilities for several of the subtopic scores are somewhat low.

The lower reliabilities of the subtopics generally were considered to be primarily a function of the restricted number of items within the subtopics; i.e., there were only four items within each subtopic. This restriction was dictated, to a large extent, by the practical exigencies of the research situation in the schools where a limited amount of testing time was available. Even though some of the subtopic scores were characterized by relatively low reliabilities, they were considered sufficiently high, for the purposes of the present study, to yield some preliminary information. However, to obtain more generalizable results, the present study would need to be replicated using subtopic tests of greater length and higher reliability.

General Procedures

Major steps involved in data collection included the activities listed below. Classes of pupils were selected and pupils eligible to be subjects were identified. Prior to the test administration, a detailed testing schedule was formulated with the cooperation of appropriate school personnel. The testing session was scheduled so that all selected classes were administered the tests at approximately the same time on May 8, 1964. Meetings were held with teachers and principals in order to orient them to the project and to provide them with materials and information regarding test schedules and testing

Table 3

Reliability Coefficients for the Mathematics Topic and Subtopic Scores[a]

	Variable	Number of Items	Grade Levels		
			Grade 4 N = 249	Grade 5 N = 267	Grade 6 N = 259
Number, Operations, and Assumptions	Y1 Addition	4	.459	.276	.387
	Y2 Subtraction	4	.466	.445	.493
	Y3 Multiplication	4	.591	.477	.622
	Y4 Division	4	.394	.467	.465
	Y5 Laws and Generalizations	4	.605	.510	.244
	Y6 Total Topic	20	.810	.696	.751
Geometry	Y7 Definition of Geometric Terms	4	.150	.088	.097
	Y8 Recognition of Geometric Figures	4	.429	.465	.708
	Y9 Measurement of Geometric Figures	4	---	.633	.622
	Y10 Total Topic	12	.421	.683	.711
Relations	Y11 Equality	4	.539	.503	.354
	Y12 Order Relationships	4	.294	.414	.558
	Y13 Maps, Graphs, and Charts	4	.315	.336	.206
	Y14 Functions	4	.140	.467	.562

Table 3 (Continued)

	Variable	Number of Items	Grade Levels		
			Grade 4 N = 249	Grade 5 N = 267	Grade 6 N = 259
	Y15 Total	16	.593	.732	.764
Numeration	Y16 Sets	4	.277	.182	.153
	Y17 Other Number Bases	4	.587	.695	.677
	Y18 Reading, Writing, Using Numbers	4	.326	.480	.569
	Y19 Roman Numerals	4	.540	.608	.644
	Y20 Total Topic	16	.724	.744	.765
Measurement	Y21 Concepts of Measurement	4	.410	.426	.488
	Y22 Conversion of Units	4	.391	.474	.612
	Y23 Operations -- No Conversion	4	.672	.704	.679
	Y24 Operations -- With Conversion	4	.331	.452	.541
	Y25 Total Topic	16	.765	.783	.830
Fractions	Y26 Concepts of Fractions	4	.473	.410	.581
	Y27 Addition of Fractions	4	.060	.533	.549
	Y28 Subtraction of Fractions	4	.328	.413	.503
	Y29 Multiplication of Fractions	4	---	---	.765

Table 3 (Continued)

	Variable	Number of Items	Grade Levels		
			Grade 4 N = 249	Grade 5 N = 267	Grade 6 N = 259
Fractions (Continued)	Y30 Division of Fractions	4	---	---	.299
	Y31 Total Topic	20	.589	.745	.700

[a] The subtopics Measurement of geometric figures, Multiplication of fractions, and Division of fractions in Grade 4, and the subtopics Multiplication of fractions and Division of fractions at grade 5 are omitted here because they were not taught at these grade levels, respectively. The scores for these subtopics were not summed for the corresponding total topic scores.

procedures. Each teacher was provided with an administrator's notebook which included a test booklet and a complete set of test directions. Following the orientation sessions, conferences were arranged for individual teachers who wished additional assistance in preparing for the test administration.

Test materials for pupils were delivered to the schools immediately prior to the test administration session and collected from the schools immediately following the test administration session. Tests taken by pupils identified as not having met the subject selection criteria were eliminated. The project's research staff did the test scoring and other work required for preparing the data for the electronic computers.

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CHAPTER III

THE VERTICAL SEQUENCES

Research Objective and Procedures for Analyses

The first objective for the study was this: to identify, at grade levels 4, 5, and 6, vertical sequences among subtopics within several mathematics topics on the basis of the criterion, level of complexity of the subtopics. Essentially, this objective involved using simplex analyses to examine the structure of intercorrelations among pupils' mathematics subtopic scores: that is, the investigators' task was to determine whether the simplex model fits sets of data ordered *a priori* on the basis of relative complexity.

The criterion for complexity was degree of inclusiveness. In the context of the present study, degree of inclusiveness refers to the extent that any one subtopic within a set of subtopics involves activities characteristic of other subtopics in that set plus additional activities. For example, a set of activities which varies in inclusiveness in terms of this type of accretion can be described in this way: subtopic #1 consists of activity *a*; subtopic #2 consists of activity *a*, and, in addition, activity *b*; subtopic #3 consists of activity *a*, activity *b*, and, in addition, activity *c*; and so on.

The first step in the analyses was to obtain the intercorrelation[1] matrix for the set of subtopics encompassed by each one of the six topics: assumptions; geometry; relations; numeration; measurement; and fractions. Examining the structure of these intercorrelations involved three more steps: ordering the subtopics; determining the relative complexity of each subtopic; and assessing the goodness of fit of the simplex model to the empirical data.

The procedure for ordering the subtopics within an intercorrelation matrix followed Guttman's (1954, 1965) approach. That is, the subtopics were ordered *a priori* using two kinds of evidence: results of a rational analysis of the relative degrees of inclusiveness of the subtopics involved; and any previously obtained data about the order among the subtopics in question.

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[1] The statistic used here was the Pearson product moment correlation coefficient.

The relative complexity of the subtopics in a set was determined by calculating the simplex loadings (A_j and a_j) for the subtopic. Procedures presented by Kaiser (1962) were used to obtain these scale values.

Kaiser's (1962) procedures also were used to assess the goodness of fit of the simplex model to the data. These procedures involved two facets: comparing the observed and the reproduced r_{jk} and R_{jk} matrices; calculating the index q^2 -- the ratio of the sum of squares accounted for by the model to the total sum of squares -- which, according to Kaiser (1962, p. 159) "(measures) the quality of the fit".

In the subsequent presentation of results, separate sections are devoted to each one of the six topics. Each section is organized by the following outline: description of the subtopics; specification of the expected order among subtopics and the rationale for this expectation; citation of the evidence -- the observed r_{jk} matrix, the observed R_{jk} matrix with accompanying A_j, a_j, and q^2 values, the reproduced R_{jk} matrix, and the reproduced r_{jk} matrix; summary comment regarding the simplex loadings and the goodness of fit of the simplex model to the data; and statement of conclusions.

Results

Number, Operations, and Assumptions

Number, operations, and assumptions included five subtopics: addition, subtraction, multiplication, division, and laws and generalizations. The instructional objectives and test items for these subtopics are on page A-11 through page A-15 of Appendix B. Mathematics activities involved are summarized below.

1. Addition

   Adding, with regrouping, two or more addends, with the addends in each problem consisting of two or more digits.

2. Subtraction

   Subtracting two, three, and four place numbers with regrouping.

3. Multiplication

   Multiplying, with regrouping, using two and three place multipliers, including zeros.

4. Division

Dividing by one and two place numbers with and without remainders and including regrouping.

5. Laws and Generalizations

Demonstrating understanding of properties of 1 and 0, addition and subtraction as inverse processes, and the commutative and associative laws.

The following simplical arrangement was expected among the subtopics: addition, subtraction, multiplication, division, and laws and generalizations. The rationale for this expectation is presented below.

Multiplication involving multipliers of two or more places requires not only multiplication *per se* but also addition and the associated regrouping process. Therefore, multiplication is more inclusive than addition alone.

Division involving divisors of more than two places and remainders necessitates both multiplication and subtraction with the associated regrouping processes as well as division, *per se*. Consequently, division is more inclusive than either subtraction or multiplication.

The inclusiveness criterion *per se* does not suggest a relationship between division and addition, between multiplication and subtraction, and between addition and subtraction. However, previous evidence presented by Guttman (1954) and Kaiser (1962) revealed the following order in the data they analyzed: from least complex to most complex -- addition, subtraction, multiplication, and division.

The laws and generalizations subtopic requires a knowledge of the four fundamental operations. In addition, this subtopic involves knowledge of specific concepts (*e.g.*, the concept of zero). As a result, laws and generalization exceeds the remaining subtopics in inclusiveness.

As stated above, the simplical order among the subtopics within the number, operations, and assumptions topic was expected to be addition, subtraction, multiplication, division, and laws and generalization. Tables 4 through 15 include evidence relevant to this expectation. In the expected hierarchy of subtopics, the simplex loadings, A_j and a_j, for the several variables become progressively larger; this is so at each grade level -- 4, 5, and 6 (Tables 5, 9, and 13). For each respective grade level, the observed and reproduced r_{jk} matrices and the observed and reproduced R_{jk} matrices are reasonably similar (Tables 4, 7, 5, 6; Tables 8, 11, 9, 10; and Tables 12, 15, 13, 14). The index q^2 is .889 for grade 4, .942 for grade 5, and .868 for grade 6 (Tables 5, 9, 13).

This evidence confirms the expectancy. At the fourth, fifth, and sixth grade levels, the simplex model fits the data reflecting the interrelations among the subtopics within the number, operations, and assumptions topic. The vertical sequence from least to most complex is the following: addition, subtraction, multiplication, division, and laws and generalizations.

Table 4

Grade 4 -- Number, Operations, and Assumptions: Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4	5
1) Addition	1.000	.233	.106	.131	.135
2) Subtraction	.233	1.000	.490	.298	.403
3) Multiplication	.106	.490	1.000	.477	.323
4) Division	.131	.298	.477	1.000	.189
5) Laws and Generalizations	.135	.403	.323	.189	1.000

Table 5

Grade 4 -- Number, Operations, and Assumptions: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	5	R_j	A_j	a_j
1)	.0000	-.6326	-.9747	-.8827	-.8697	-3.3597	-.6719	.2129
2)	.6326	.0000	-.3098	-.5258	-.3947	- .5977	-.1195	.7595
3)	.9747	.3098	.0000	-.3215	-.4908	.4722	.0944	1.2429
4)	.8827	.5258	.3215	.0000	-.7235	1.0065	.2013	1.5896
5)	.8697	.3947	.4908	.7235	.0000	2.4787	.4957	3.1314
	$\Sigma R_j^2 = 19.0248$			$n\Sigma\Sigma R_{jk}^2 = 21.4085$			$q^2 = .889$	

Table 6

Grade 4 -- Number, Operations, and Assumptions: Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4	5
1) Addition	.0000	-.5524	-.7663	-.8732	-1.1676
2) Subtraction	.5524	.0000	-.2139	-.3208	-.6152
3) Multiplication	.7663	.2139	.0000	-.1069	-.4013
4) Division	.8732	.3208	.1069	.0000	-.2944
5) Laws and Generalizations	1.1676	.6152	.4013	.2944	.0000

Table 7

Grade 4 -- Number, Operations, and Assumptions: Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4	5
1) Addition	1.0000	.2803	.1713	.1339	.0680
2) Subtraction	.2803	.0000	.6111	.4778	.2425
3) Multiplication	.1713	.6111	.0000	.7819	.3969
4) Division	.1339	.4778	.7819	.0000	.5076
5) Laws and Generalizations	.0680	.2425	.3969	.5076	.0000

Table 8

Grade 5 -- Number, Operations, and Assumptions: Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4	5
1) Addition	1.000	.249	.233	.133	.111
2) Subtraction	.249	1.000	.459	.335	.285
3) Multiplication	.233	.459	1.000	.503	.354
4) Division	.133	.335	.503	1.000	.239
5) Laws and Generalizations	.111	.285	.354	.239	1.000

Table 9

Grade 5 -- Number, Operations, and Assumptions: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	5	R_j	A_j	a_j
1)	.0000	-.6038	-.6326	-.8761	-.9547	-3.0672	-.6134	.2436
2)	.6038	.0000	-.3382	-.4750	-.5452	- .7546	-.1509	.7065
3)	.6326	.3382	.0000	-.2984	-.4510	.2214	.0443	1.1074
4)	.8761	.4750	.2984	.0000	-.6216	1.0279	.2056	1.6056
5)	.9547	.5452	.4510	.6216	.0000	2.5725	.5145	3.2700

$\Sigma R_j^2 = 17.7005$ $n\Sigma\Sigma R_{jk}^2 = 18.7991$ $q^2 = .942$

Table 10

Grade 5 -- Number, Operations, and Assumptions: Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4	5
1) Addition	.0000	-.4625	-.6577	-.8190	-1.1279
2) Subtraction	.4625	.0000	-.1952	-.3565	- .6654
3) Multiplication	.4625	.1952	.0000	-.1613	- .4702
4) Division	.6577	.3565	.1613	.0000	- .3089
5) Laws and Generalizations	.8190	.6654	.4702	.3089	.0000

Table 11

Grade 5 -- Number, Operations, and Assumptions: Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4	5
1) Addition	.0000	.3448	.2200	.1517	.0745
2) Subtraction	.3448	.0000	.6380	.4400	.2161
3) Multiplication	.2200	.6380	.0000	.6897	.3387
4) Division	.1517	.4400	.6897	.0000	.4910
5) Laws and Generalizations	.0745	.2161	.3387	.4910	.0000

Table 12

Grade 6 -- Number, Operations, and Assumptions:
Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4	5
1) Addition	1.000	.152	.139	.246	.157
2) Subtraction	.152	1.000	.416	.346	.210
3) Multiplication	.139	.416	1.000	.422	.338
4) Division	.246	.346	.422	1.000	.196
5) Laws and Generalizations	.157	.210	.338	.196	1.000

Table 13

Grade 6 -- Number, Operations, and Assumptions:
Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Subtopics	1	2	3	4	5	R_j	A_j	a_j
1)	.0000	-.8182	-.8570	-.6091	-.8041	-3.0884	-.6177	.2412
2)	.8182	.0000	-.3809	-.4609	-.6778	- .7014	-.1403	.7240
3)	.8570	.3809	.0000	-.3747	-.4711	.3921	.0784	1.1978
4)	.6091	.4609	.3747	.0000	-.7066	.7381	.1476	1.4048
5)	.8041	.6778	.4711	.7066	.0000	2.6596	.5319	3.4031

$\Sigma R_j^2 = 17.8022$ $\quad n\Sigma\Sigma R_{jk}^2 = 20.5001$ $\quad q^2 = .868$

Table 14

Grade 6 -- Number, Operations, and Assumptions: Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4	5
1) Addition	.0000	-.4774	-.6961	-.7653	-1.1496
2) Subtraction	.4774	.0000	-.2187	-.2879	- .6722
3) Multiplication	.6961	.2187	.0000	-.0692	- .4535
4) Division	.7653	.2879	.0692	.0000	- .3843
5) Laws and Generalizations	1.1496	.6722	.4535	.3843	.0000

Table 15

Grade 6 -- Number, Operations, and Assumptions: Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4	5
1) Addition	.0000	.3331	.2014	.1717	.0708
2) Subtraction	.3331	.0000	.6044	.5154	.2127
3) Multiplication	.2014	.6044	.0000	.8526	.3520
4) Division	.1717	.5154	.8526	.0000	.4128
5) Laws and Generalizations	.0708	.2127	.3520	.4128	.0000

Geometry

Geometry consisted of three subtopics: definitions of geometric terms, recognition of geometric figures, and measurement of geometric figures. The instructional objectives and test items for these subtopics are presented in Appendix B on page A16 through page A18. Mathematics activities[1] sampled are summarized below.

1. Definitions of Geometric Terms

   Defining terms referring to parallel lines and angles, circles, and sets of points.

2. Recognition of Geometric Figures

   Recognizing and naming two dimensional (plane) figures and three dimensional (solid) figures.

3. Measurement of Geometric Figures

   Finding the perimeter and area of various polygons and the volume of various polyhedrons.

Among the subtopics, the following simplical arrangement was expected: definitions of geometric terms, recognition of geometric figures, and measurement of geometric figures. The rationale for this expectation is presented below.

Definitions of geometric terms is the least inclusive of the three subtopics. Involved in this subtopic is knowledge of labels used to refer to geometry content.

Recognition of geometric figures involves knowledge of geometry terminology. Required in addition are knowledge of the characteristics of a variety of geometric figures and the ability to distinguish among figures which have a number of similar features.

Measurement of geometric figures requires knowledge of geometry terminology and facility in recognizing geometric figures. In addition, measurement of geometric figures necessitates other kinds of knowledge: knowledge about which operations are required to compute the various measures (e.g., area), facility in performing the required operations

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[1] The fourth grade instructional program did not include some of the listed mathematics activities. Consequently, the material presented in this section pertains primarily to the fifth and sixth grade levels.

(e.g., multiplication), and knowledge about measurement concepts, terms, and conventions (e.g., width, cu. in.).

In sum, the expected simplical arrangement among the geometry subtopics was the following: definitions of geometric terms, recognition of geometric figures, and measurement of geometric figures. Tables 16 through 23 consist of information pertinent to this expectation. In the expected hierarchy of subtopics, the simplex loadings, A_j and a_j, for the several variables become progressively larger; this is so at each grade level -- 5 and 6 (Tables 17, 21). For each respective grade level, the observed and reproduced r_{jk} matrices and the observed and reproduced R_{jk} matrices are generally similar (Tables 16, 19, 17, 18; Tables 20, 23, 21, 22). The index q^2 is .920 for grade 5 and .953 for grade 6 (Tables 17, 21).

This evidence confirms the expectancy. At the fifth and sixth grade levels, the simplex model fits the data reflecting the interrelations among the subtopics within the geometry topic. The vertical sequence from least to most complex is the following: definitions of geometric terms, recognition of geometric figures, and measurement of geometric figures.

Table 16

Grade 5 -- Geometry:
Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3
1) Definitions of Geometric Terms	1.000	.334	.299
2) Recognition of Geometric Figures	.334	1.000	.346
3) Measurement of Geometric Figures	.299	.346	1.000

Table 17

Grade 5 -- Geometry:
Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Subtopics	1	2	3	R_j	A_j	a_j
1)	.0000	-.4763	-.5243	-1.0006	-.3335	.4640
2)	.4763	.0000	-.4609	.0154	.0051	1.0119
3)	.5243	.4609	.0000	.9852	.3284	2.1300

$\Sigma R_j^2 = 1.9721$ $n\Sigma\Sigma R_{jk}^2 = 2.1425$ $q^2 = .920$

Table 18

Grade 5 -- Geometry:
Matrix of ($A_j - A_k$), the Reproduced R_{jk}

Subtopics	1	2	3
1) Definitions of Geometric Terms	.0000	-.3386	-.6619
2) Recognition of Geometric Figures	.3386	.0000	-.3233
3) Measurement of Geometric Figures	.6619	.3233	.0000

Table 19

Grade 5 -- Geometry:
Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3
1) Definitions of Geometric Terms	.0000	.4585	.2178
2) Recognition of Geometric Figures	.4585	.0000	.4751
3) Measurement of Geometric Figures	.2178	.4751	.0000

Table 20

Grade 6 -- Geometry: Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3
1) Definitions of Geometric Terms	1.000	.362	.296
2) Recognition of Geometric Figures	.362	1.000	.414
3) Measurement of Geometric Figures	.296	.414	1.000

Table 21

Grade 6 -- Geometry: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Subtopics	1	2	3	R_j	A_j	a_j
1)	.0000	-.4413	-.5287	-.9700	-.3233	.4750
2)	.4413	.0000	-.3805	.0583	.0194	1.0457
3)	.5287	.3830	.0000	.9117	.3039	2.0132

$\Sigma R_j^2 = 1.7755$ $\quad n\Sigma\Sigma R_{jk}^2 = 1.8629$ $\quad q^2 = .953$

Table 22

Grade 6 -- Geometry:
Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3
1) Definitions of Geometric Terms	.0000	-.3427	-.6272
2) Recognition of Geometric Figures	.3427	.0000	-.2845
3) Measurement of Geometric Figures	.6272	.2845	.0000

Table 23

Grade 6 -- Geometry:
Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3
1) Definitions of Geometric Terms	.0000	.4542	.2359
2) Recognition of Geometric Figures	.4542	.0000	.5194
3) Measurement of Geometric Figures	.2359	.5194	.0000

Relations

Four topics are subsumed under relations: namely, equality; order relations; maps, graphs, and charts; and functions. Appendix B (pages A19 through A23) contains the instructional objectives and test items. Mathematics activities sampled are summarized below.

1. Equality

   Demonstrating knowledge of symbols for and understanding of concepts of equivalence and equality relations among numbers, series of numbers, and sizes.

2. Order Relations

   Demonstrating understanding of order relations involving direction and distance concepts, abstractly and concretely expressed time concepts illustrated with number lines, and place value involving whole numbers and decimal fractions.

3. Maps, Graphs, Charts

   Reading and interpreting simple bar graphs.

4. Functions

   Using concepts of ratio and proportion in solving problems related to quantity, distance, money, and time.

The following simplical arrangement was expected among the subtopics: maps, graphs, and charts; equality; functions; and order relations. The rationale for this expectation is presented below.

The maps, graphs, and charts subtopic involves demonstrating understanding of equivalence and equality relations by interpreting relations portrayed on a simple bar graph. This subtopic would seem to be the least complex of the four subtopics.

Equality also requires understanding of concepts of equivalence and equality. In addition, this subtopic necessitates recognizing symbols for equality and equivalence relations as well as applying the equivalence and equality concepts in answering questions involving both measurements and numbers.

Functions parallels the preceding two subtopics in requiring understanding of the equivalence and equality concepts. In addition, activities in the functions subtopic necessitate applying the ratio and proportion facets of the equality and equivalence concepts to a variety of measurements.

Order relations is the most complex of all. This subtopic includes not only the basic equivalence and equality relations, but also use of the number line, reasoning with abstractly and concretely presented material, and knowledge of place value in relation to whole numbers and decimal fractions.

To recapitulate, the expected simplical arrangement of the relations subtopics was the following: maps, graphs, and charts; equality; functions; and order relations. Tables 24 through 35 contain evidence related to this expectation. In the expected hierarchy of subtopics, the simplex loadings, A_j and a_j, for the several variables become progressively larger; this is so at each grade level -- 4, 5, and 6 (Tables 25, 29, 33). For each respective grade level, the observed and reproduced r_{jk} matrices and the observed and reproduced R_{jk} matrices are quite similar (Tables 24, 27, 25, 26; Tables 28, 31, 29, 30; Tables 32, 35, 33, 34). The index q^2 is .919 for grade 4, .896 for grade 5, and .834 for grade 6 (Tables 25, 29, 33).

This evidence confirms the expectancy. At the fourth, fifth, and sixth grade levels, the simplex model fits the data reflecting the interrelations among the subtopics within the relations topic. The vertical sequence from least to most complex is the following: maps, graphs, and charts; equality; functions; and order relations.

Table 24

Grade 4 -- Relations:
Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	1.000	.365	.297	.128
2) Equality	.365	1.000	.239	.285
3) Functions	.297	.239	1.000	.276
4) Order Relations	.128	.285	.276	1.000

Table 25

Grade 4 -- Relations:
Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	R_j	A_j	a_j
1)	.0000	-.4377	-.5272	-.8928	-1.8577	-.4644	.3432
2)	.4377	.0000	-.6216	-.5452	- .7291	-.1823	.6572
3)	.5272	.6216	.0000	-.5591	.5897	.1474	1.4042
4)	.8928	.5452	.5591	.0000	1.9971	.4993	3.1571
	$\Sigma R_j^2 = 8.3188$			$n\Sigma\Sigma R_{jk}^2 = 9.0513$		$q^2 = .919$	

Table 26

Grade 4 -- Relations: Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	.0000	-.2821	-.6118	-.9637
2) Equality	.2821	.0000	-.3297	-.6816
3) Functions	.6118	.3297	.0000	-.3519
4) Order Relations	.9137	.6816	.3519	.0000

Table 27

Grade 4 -- Relations: Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	.0000	.5222	.2444	.1087
2) Equality	.5222	.0000	.4680	.2082
3) Functions	.2444	.4680	.0000	.4448
4) Order Relations	.1087	.2082	.4448	.0000

Table 28

Grade 5 -- Relations: Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	1.000	.338	.297	.256
2) Equality	.338	1.000	.278	.318
3) Functions	.297	.278	1.000	.451
4) Order Relations	.256	.318	.451	1.000

Table 29

Grade 5 -- Relations: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	R_j	A_j	a_j
1)	.0000	-.4711	-.5272	-.5918	-1.5901	-.3975	.4004
2)	.4711	.0000	-.5560	-.4976	- .5825	-.1456	.7152
3)	.5272	.5560	.0000	-.3458	.7374	.1843	1.5286
4)	.5918	.4976	.3458	.0000	1.4352	.3588	2.2847
	$\Sigma R_j^2 = 5.4713$			$n\Sigma\Sigma R_{jk}^2 = 6.1057$		$q^2 = .896$	

Table 30

Grade 5 -- Relations: Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	.0000	-.2519	-.5818	-.7563
2) Equality	.2519	.0000	-.3299	-.5044
3) Functions	.5818	.3299	.0000	-.1745
4) Order Relations	.7563	.5044	.1745	.0000

Table 31

Grade 5 -- Relations: Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	.0000	.5598	.2619	.1753
2) Equality	.5598	.0000	.4679	.3130
3) Functions	.2619	.4679	.0000	.6691
4) Order Relations	.1753	.3130	.6691	.0000

Table 32

Grade 6 -- Relations:
Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	1.000	.260	.302	.321
2) Equality	.260	1.000	.319	.433
3) Functions	.302	.319	1.000	.443
4) Order Relations	.321	.433	.443	1.000

Table 33

Grade 6 -- Relations:
Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	R_j	A_j	a_j
1)	.0000	-.5850	-.5200	-.4935	-1.5985	-.3996	.3985
2)	.5850	.0000	-.4962	-.3635	- .2747	-.0687	.8537
3)	.5200	.4962	.0000	-.3536	.6626	.1657	1.4645
4)	.4935	.3635	.3536	.0000	1.2106	.3027	2.0077
	$\Sigma R_j^2 = 4.5353$			$n\Sigma\Sigma R_{jk}^2 = 5.4382$		$q^2 = .834$	

Table 34

Grade 6 -- Relations:
Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	.0000	-.3309	-.5653	-.7023
2) Equality	.3309	.0000	-.2344	-.3714
3) Functions	.5653	.2344	.0000	-.1370
4) Order Relations	.7023	.3714	.1370	.0000

Table 35

Grade 6 -- Relations:
Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Maps, Graphs, and Charts	.0000	.4668	.2721	.1985
2) Equality	.4668	.0000	.5829	.4252
3) Functions	.2721	.5829	.0000	.7294
4) Order Relations	.1985	.4252	.7294	.0000

Numeration

Numeration was composed of four subtopics: sets; other numeration bases; reading, writing, and using whole numbers and decimals; and Roman numerals. The instructional objectives and test items for these subtopics appear in Appendix B on pages A24 through A27. Summarized below are mathematics activities sampled.

1. Sets

   Demonstrating an understanding of the concept of sets and knowledge of set terminology, distinguishing among kinds of sets, and demonstrating an understanding of the set operations -- union and intersection.

2. Other Numeration Bases

   Comparing base five and base ten, counting beyond the square of the base, and using other numeration bases than base ten.

3. Reading, Writing, and Using Whole Numbers and Decimals

   Demonstrating understanding of place value by reading, writing, and using whole numbers and decimals involving multiple places.

4. Roman Numerals

   Reading and writing Roman numerals corresponding to Arabic numerals through 80.

Among the subtopics, the following simplical arrangement was expected: Roman numerals; reading, writing, and using whole numbers and decimals; other numeration bases; and sets. The rationale for this expectation is presented below.

The activities involved in Roman numerals primarily require knowledge of the correspondence of the symbols in the Roman and Arabic numeration systems; also, activities used herein necessitated knowledge of addition facts to 20.

Reading, writing, and using whole numbers and decimals is more inclusive. This subtopic requires knowledge of Arabic numeration symbols for whole numbers and rational numbers. Also, the subtopic necessitates understanding of and ability to apply the concept of place value in the base ten.

Other numeration bases becomes more complex. This subtopic involves knowledge of Arabic numeration symbols, understanding of the concept of numeration bases as well as specifics of several numeration bases, understanding of the concept of place value per se, and knowledge of specifics of place value in relation to several numeration bases other than base ten.

Sets is even more inclusive. Not only does sets require knowledge of terminology and concepts related to such topics as Arabic numeration symbols and knowledge of relations such as equality, equivalence, and redundancy; but also sets involves knowledge of set terminology as well as understanding of and ability to use concepts and operations related to sets as a more general organizational system applicable to any numerical or other domain.

As stated above, the following simplical arrangement among the numeration subtopics was expected: Roman numerals; reading, writing, and using whole numbers and decimals; other numeration bases; and sets. Tables 36 through 47 are composed of information pertaining to this expectation. In the expected hierarchy of subtopics, the simplex loadings, A_j and a_j, for the several variables become progressively larger; this is so at each grade level -- 4, 5, and 6 (Tables 37, 41, 45). For each respective grade level, the observed and reproduced r_{jk} matrices and the observed and reproduced R_{jk} matrices are fairly similar (Tables 36, 39, 37, 38; Tables 40, 43, 41, 42; Tables 44, 47, 45, 46). The index q^2 is .909 for grade 4, .864 for grade 5, and .926 for grade 6 (Tables 37, 41, 45).

This evidence confirms the expectancy. At the fourth, fifth, and sixth grade levels, the simplex model fits the data reflecting the interrelations among the subtopics within the numeration topic. The vertical sequence from least to most complex is the following: Roman numerals; reading, writing, and using whole numbers and decimals; other numeration bases; and sets.

Table 36

Grade 4 -- Numeration:
Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Roman Numerals	1.000	.432	.279	.259
2) Reading, Writing, and Using Whole Numbers and Decimals	.432	1.000	.338	.426
3) Other Numeration Bases	.279	.338	1.000	.406
4) Sets	.259	.426	.406	1.000

Table 37

Grade 4 -- Numeration:
Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	R_j	A_j	a_j
1)	.0000	-.3645	-.5544	-.5867	-1.5056	-.3764	.421
2)	.3645	.0000	-.4711	-.3706	- .4772	-.1193	.760
3)	.5544	.4711	.0000	-.3915	.6340	.1585	1.144
4)	.5867	.3706	.3915	.0000	1.3488	.3372	1.217

$\Sigma R_j^2 = 4.7158$ $n\Sigma\Sigma R_{jk}^2 = 5.1880$ $q^2 = .909$

Table 38

Grade 4 -- Numeration: Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Roman Numerals	.0000	-.2571	-.5349	-.7136
2) Reading, Writing, and Using Whole Numbers and Decimals	.2571	.0000	-.2778	-.4565
3) Other Numeration Bases	.5349	.2778	.0000	-.1787
4) Sets	.7136	.4565	.1787	.0000

Table 39

Grade 4 -- Numeration: Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Roman Numerals	1.0000	.5539	.3680	.3459
2) Reading, Writing, and Using Whole Numbers and Decimals	.5539	1.0000	.6643	.6248
3) Other Numeration Bases	.3680	.6643	1.0000	.9400
4) Sets	.3459	.6248	.9400	1.0000

Table 40

Grade 5 -- Numeration: Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Roman Numerals	1.000	.410	.486	.280
2) Reading, Writing, and Using Whole Numbers and Decimals	.410	1.000	.390	.263
3) Other Numeration Bases	.486	.390	1.000	.222
4) Sets	.280	.263	.222	1.000

Table 41

Grade 5 -- Numeration: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	R_j	A_j	a_j
1)	.0000	-.3872	-.3134	-.5528	-1.2534	-.3133	.4861
2)	.3872	.0000	-.4089	-.5800	- .6017	-.1504	.7073
3)	.3134	.4089	.0000	-.6536	.0687	.0172	1.0405
4)	.5528	.5800	.6536	.0000	1.7864	.4466	2.7963

$\Sigma R_j^2 = 5.1290$ $n\Sigma\Sigma R_{jk}^2 = 5.9381$ $q^2 = .864$

Table 42

Grade 5 -- Numeration: Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Roman Numerals	.0000	-.1629	-.3305	-.7599
2) Reading, Writing, and Using Whole Numbers and Decimals	.1629	.0000	-.1676	-.5970
3) Other Numeration Bases	.3305	.1676	.0000	-.4294
4) Sets	.7599	.5970	.4294	.0000

Table 43

Grade 5 -- Numeration: Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Roman Numerals	.0000	.6873	.4672	.1738
2) Reading, Writing, and Using Whole Numbers and Decimals	.6873	.0000	.6798	.2529
3) Other Numeration Bases	.4672	.6798	.0000	.3721
4) Sets	.1738	.2529	.3721	.0000

Table 44

Grade 6 -- Numeration: Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Roman Numerals	1.000	.422	.305	.185
2) Reading, Writing, and Using Whole Numbers and Decimals	.422	1.000	.335	.118
3) Other Numeration Bases	.305	.335	1.000	.193
4) Sets	.185	.118	.193	1.000

Table 45

Grade 6 -- Numeration: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Subtopics	1	2	3	4	R_j	A_j	a_j
1)	.0000	-.3747	-.5157	-.7328	-1.6232	-.4058	.3928
2)	.3747	.0000	-.4750	-.9281	-1.0284	-.2571	.5533
3)	.5157	.4750	.0000	.7144	.2763	.0691	1.1724
4)	.7328	.9281	.7144	.0000	2.3753	.5938	3.9245

$\Sigma R_j^2 = 9.4108$ $n\Sigma\Sigma R_{jk}^2 = 10.1628$ $q^2 = .926$

Table 46

Grade 6 -- Numeration: Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Roman Numerals	.0000	-.1487	-.4749	-.9996
2) Reading, Writing, and Using Whole Numbers and Decimals	.1487	.0000	-.3262	-.8509
3) Other Numeration Bases	.4749	.3262	.0000	-.5247
4) Sets	.9996	.8509	.5247	.0000

Table 47

Grade 6 -- Numeration: Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Roman Numerals	.0000	.7099	.3350	.1001
2) Reading, Writing, and Using Whole Numbers and Decimals	.7099	.0000	.4719	.1410
3) Other Numeration Bases	.3350	.4719	.0000	.2987
4) Sets	.1001	.1410	.2987	.0000

Measurement

Measurement encompassed four subtopics: measurement concepts, conversion of units, operations without conversion, and operations with conversion. Pages A28 through A31 of Appendix B contain the instructional objectives and test items. Mathematical activities sampled are summarized below.

1. **Measurement Concepts**

   Demonstrating a knowledge of measurement terminology and an understanding of concepts related to size, quantity, distance, time, and weight.

2. **Conversion of Units**

   Demonstrating ability to change, from one unit to another, measures related to size, distance, time, and money.

3. **Operations without Conversion**

   Using the fundamental operations, addition, subtraction, etc., in solving problems involving measures which are expressed in a common unit.

4. **Operations with Conversion**

   Using the fundamental operations, addition, subtraction, etc., in solving problems involving measures which are expressed in different units and which require conversion to a common unit.

Among the subtopics, the following simplical arrangement was expected: measurement concepts, operations without conversion, conversion of units, and operations with conversion. The rationale for this expectation is presented below.

Of the four subtopics, concepts of measurement is the least complex. It primarily involves a knowledge of terms and conventions for the various measurement units and a knowledge of which specific units belong in various categories of measurement.

Operations with no conversion requires the same knowledge of measurement terminology and concepts so that decisions can be made about the need to convert units before a problem can be solved. Since no conversions actually were needed for the problems used herein, the only additional activity involved deciding what fundamental arithmetic operations were appropriate for solving the problems and facility in performing those operations.

Conversion of units is more complex. Knowledge of measurement terminology and concepts and ability to perform the fundamental operations are involved. Required in addition are the following: knowledge about the size of each unit, knowledge about the relative sizes of the units, and ability to decide what operations are needed to make the required conversions.

Operations with conversion requires activities involved in the three preceding subtopics. In addition, it necessitates the ability to decide what kind of conversion is needed for the problem in question as well as the ability to decide what operation is required to solve the problem after the conversion is made.

To recapitulate, the expected simplical arrangement among the measurement subtopics was this one: measurement concepts, operations without conversion, conversion of units, and operations with conversion. Tables 48 through 59 encompass data germane to this expectation. In the expected hierarchy of subtopics, the simplex loadings, A_j and a_j, for the several variables become progressively larger; this is so at each grade level -- 4, 5, and 6 (Tables 49, 53, 57). For each respective grade level, the observed and reproduced r_{jk} matrices and the observed and reproduced R_{jk} matrices are generally similar (Tables 48, 51, 49, 50; Tables 52, 55, 53, 54; Tables 56, 59, 57, 58). The index q^2 is .871 for grade 4, .833 for grade 5, and .702 for grade 6 (Tables 49, 53, 57).

This evidence confirms the expectancy. At the fourth, fifth, and sixth grade levels, the simplex model fits the data reflecting the interrelations among the subtopics within the measurement topic. The vertical sequence from least to most complex is the following: concepts of measurement, operations without conversion, conversion of units, and operations with conversion.

Table 48

Grade 4 -- Measurement:
Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Concepts of Measurement	1.000	.336	.309	.357
2) Operations Without Conversion	.336	1.000	.491	.342
3) Conversion of Units	.309	.491	1.000	.392
4) Operations With Conversion	.357	.342	.392	1.000

Table 49

Grade 4 -- Measurement:
Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	R_j	A_j	a_j
1)	.0000	-.4737	-.5100	-.4473	-1.4310	-.3578	.4387
2)	.4737	.0000	-.3089	-.4660	- .3012	-.0753	.8408
3)	.5100	.3089	.0000	-.4067	.4122	.1031	1.2679
4)	.4473	.4660	.4067	.0000	1.3200	.3300	2.1380

$\Sigma R_j^2 = 4.0508$ $\quad n\Sigma\Sigma R_{jk}^2 = 4.6502$ $\quad q^2 = .871$

Table 50

Grade 4 -- Measurement:
Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Concepts of Measurement	.0000	-.2825	-.4609	-.6878
2) Operations Without Conversion	.2825	.0000	-.1784	-.4053
3) Conversion of Units	.4609	.1784	.0000	-.2269
4) Operations With Conversion	.6878	.4053	.2269	.0000

Table 51

Grade 4 -- Measurement:
Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Concepts of Measurement	.0000	.5218	.3460	.2052
2) Operations Without Conversion	.5218	.0000	.6631	.3933
3) Conversion of Units	.3460	.6631	.0000	.5930
4) Operations With Conversion	.2052	.3933	.5930	.0000

Table 52

Grade 5 -- Measurement: Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Concepts of Measurement	1.000	.411	.428	.426
2) Operations Without Conversion	.410	1.000	.395	.398
3) Conversion of Units	.428	.395	1.000	.466
4) Operations With Conversion	.426	.398	.466	1.000

Table 53

Grade 5 -- Measurement: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	R_j	A_j	a_j
1)	.0000	-.3862	-.3686	-.3706	-1.1254	-.2813	.5233
2)	.3862	.0000	-.4034	-.4001	- .4173	-.1043	.7865
3)	.3686	.4034	.0000	-.3316	.4404	.1101	1.2885
4)	.3706	.4001	.3316	.0000	1.1023	.2756	1.8861
	$\Sigma R_j^2 = 2.8497$			$n\Sigma\Sigma R_{jk}^2 = 3.4205$		$q^2 = .833$	

Table 54

Grade 5 -- Measurement:
Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Concepts of Measurement	.0000	-.1770	-.3914	-.5569
2) Operations Without Conversion	.1770	.0000	-.2144	-.3799
3) Conversion of Units	.3914	.2144	.0000	-.1655
4) Operations With Conversion	.5569	.3799	.1655	.0000

Table 55

Grade 5 -- Measurement:
Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Concepts of Measurement	.0000	.6654	.4061	.2775
2) Operations Without Conversion	.6654	.0000	.6104	.4170
3) Conversion of Units	.4061	.6104	.0000	.6832
4) Operations With Conversion	.2775	.4170	.6832	.0000

Table 56

Grade 6 -- Measurement: Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4
1) Concepts of Measurement	1.000	.380	.496	.604
2) Operations Without Conversion	.380	1.000	.378	.470
3) Conversion of Units	.496	.378	1.000	.578
4) Operations With Conversion	.604	.470	.578	1.000

Table 57

Grade 6 -- Measurement: Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	R_j	A_j	a_j
1)	1.0000	-.4202	-.3036	-.2190	-.9428	-.2357	.581
2)	.4202	.0000	-.4225	-.3279	-.3302	-.0825	.827
3)	.3036	.4225	.0000	-.2381	.4880	.1220	1.325
4)	.2190	.3279	1.2381	.0000	.7850	.1963	1.572

$\Sigma R_j^2 = 1.8523$ $n\Sigma\Sigma R_{jk}^2 = 2.6376$ $q^2 = .702$

Table 58

Grade 6 -- Measurement:
Matrix of (A_j - A_k), the Reproduced R_{jk}

Subtopics	1	2	3	4
1) Concepts of Measurement	.0000	-.1532	-.3577	-.4320
2) Operations Without Conversion	.1532	.0000	-.2045	-.2788
3) Conversion of Units	.3577	.2045	.0000	-.0743
4) Operations With Conversion	.4320	.2788	.0743	.0000

Table 59

Grade 6 -- Measurement:
Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4
1) Concepts of Measurement	1.0000	.7025	.4385	.3696
2) Operations Without Conversion	.7025	1.0000	.6242	.5261
3) Conversion of Units	.4385	.6242	1.0000	.8429
4) Operations With Conversion	.3696	.5261	.8429	1.0000

Fractions

Fractions included five subtopics: concepts of fractions, addition of fractions, subtraction of fractions, multiplication of fractions, and division of fractions. Pages A32 through A36 of Appendix B contain the instructional objectives and test items. Mathematics activities[1] involved in each subtopic are summarized below.

1. Concepts of Fractions

   Recognizing fractional parts of wholes, reciprocals, and relationships between common fractions and decimals.

2. Addition of Fractions

   Adding decimals, adding like fractions, and adding unlike but related fractions.

3. Subtraction of Fractions

   Subtracting decimals and subtracting like fractions, unlike but related fractions, and fractions from whole numbers.

4. Multiplication of Fractions

   Multiplying a decimal by a decimal, a decimal by a whole number, a common fraction by a whole number, and unlike and unrelated fractions.

5. Division of Fractions

   Dividing a whole number by a decimal and a fraction by a fraction.

Among the subtopics, the following simplical arrangement was expected: concepts, addition, subtraction, multiplication, and division. The rationale for this expectation is stated below.

Concepts of fractions includes understanding of the meaning of rational numbers expressed as common fractions and decimal fractions.

In addition to this understanding, addition of fractions involves knowledge of the processes required to add fractions and decimals, ability to add *per se*, and knowledge about how to find the LCD for unlike but related fractions.

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[1] The fourth and fifth grade instructional programs did not include some of the listed mathematics activities. Consequently, the material presented in this section pertains primarily to the sixth grade level.

Subtraction of fractions requires understanding of the concept of fractions and decimals, knowledge of the processes used to subtract decimals and common fractions, ability to subtract per se, and knowledge about how to find the LCD for related fractions. In these activities, addition and subtraction of fractions are parallel activities. However, the test items used herein to sample the subtraction subtopic encompassed the additional activity of subtracting a fraction from a whole number.

Multiplication of fractions parallels subtraction of fractions in involving understanding of concepts of fractions, etc. However, multiplication becomes more complex in terms of requiring the following: knowledge about the roles of both the numerators and denominators in multiplying common fractions, and considerations related to place value in multiplying decimal fractions. In addition, the test items also included a problem which necessitated finding the LCD for unlike and unrelated fractions.

Division of fractions parallels multiplication in the types of knowledge and understanding required. However, an additional activity increases inclusiveness: viz., knowledge about the role of the reciprocal in the division of fractions.

As stated above: the subtopics subsumed under the fractions topic were expected to be characterized by the following simplical arrangement: concepts of fractions, addition of fractions, subtraction of fractions, multiplication of fractions, and division of fractions. Tables 60 through 63 contain evidence relevant to this expectation. In the expected hierarchy of subtopics at grade 6, the simplex loadings, A_j and a_j, for the several variables become progressively larger (Table 61). The observed and reproduced r_{jk} matrices and the expected and reproduced R_{jk} matrices are reasonably similar (Tables 60, 61, 62, 63). The index q^2 is .905 (Table 61).

This evidence confirms the expectancy. At the sixth grade level, the simplex model fits the data reflecting the interrelations among the subtopics within the fractions topic. The vertical sequence from least to most complex is the following: concepts of fractions, addition of fractions, subtraction of fractions, multiplication of fractions, and division of fractions.

Table 60

Grade 6 -- Fractions:
Matrix of Observed r_{jk}, Subtopics in the Expected Order

Subtopics	1	2	3	4	5
1) Concepts of Fractions	1.000	.531	.519	.451	.374
2) Addition of Fractions	.531	1.000	.558	.577	.315
3) Subtraction of Fractions	.519	.558	1.000	.535	.304
4) Multiplication of Fractions	.451	.577	.535	1.000	.421
5) Division of Fractions	.374	.315	.304	.421	1.000

Table 61

Grade 6 -- Fractions:
Matrix of Observed R_{jk} and the A_j, a_j, and q^2 Values

Sub-topics	1	2	3	4	5	R_j	A_j	a_j
1)	.0000	-.2848	-.2749	-.3458	-.4271	-1.3326	-.2665	.5414
2)	.2848	.0000	-.2534	-.2716	-.5171	- .7573	-.1515	.7055
3)	.2749	.2534	.0000	-.2388	-.5017	- .2122	-.0424	.9070
4)	.3458	.2716	.2388	.0000	-.3757	.4085	.0961	1.2477
5)	.4271	.5171	.5017	.3757	.0000	1.8216	.3643	2.3137
	$\Sigma R_j^2 = 5.9435$			$n\Sigma\Sigma R_{jk}^2 = 6.5696$		$q^2 = .905$		

Table 62

Grade 6 -- Fractions:
Matrix of $(A_j - A_k)$, the Reproduced R_{jk}

Subtopics	1	2	3	4	5
1) Concepts of Fractions	.0000	-.1150	-.2241	-.3626	-.6308
2) Addition of Fractions	.1150	.0000	-.1091	-.2476	-.5158
3) Subtraction of Fractions	.2241	.1091	.0000	-.1385	-.4067
4) Multiplication of Fractions	.3626	.2476	.1385	.0000	-.2682
5) Division of Fractions	.6308	.5158	.4067	.2682	.0000

Table 63

Grade 6 -- Fractions:
Matrix of (a_j/a_k), the Reproduced r_{jk}

Subtopics	1	2	3	4	5
1) Concepts of Fractions	.0000	.7674	.5969	.4339	.2340
2) Addition of Fractions	.7674	.0000	.7778	.5654	.3049
3) Subtraction of Fractions	.5969	.7778	.0000	.7269	.3920
4) Multiplication of Fractions	.4339	.5654	.7269	.0000	.5393
5) Division of Fractions	.2340	.3049	.3920	.5393	.0000

Summary of Conclusions

The research objective relevant to vertical sequences was this: to identify, at grade levels 4, 5, and 6, vertical sequences among subtopics within several mathematics topics on the basis of the criterion, level of complexity. Several conclusions were supported by data obtained by analyzing intercorrelations among the subtopic scores to determine whether the simplex model fitted data ordered *a priori* on the basis of the complexity criterion. These conclusions are summarized below.

1. At each grade level, a vertical hierarchy along the complexity dimension is present among the subtopics within the topics analyzed.
2. Within the respective topics, the hierarchial orders among subtopics from least to most complex are the following.
    a. Number, operations, assumptions: addition, subtraction, multiplication, division, and laws and generalizations.
    b. Geometry (fifth and sixth grade levels only)[1]: definitions of geometric terms, recognition of geometric figures, and measurement of geometric figures.
    c. Relations: maps, graphs, and charts; equality; functions; and order relations.
    d. Numeration: Roman numerals; reading, writing, and using whole numbers and decimals; other numeration bases; and sets.
    e. Measurement: concepts of measurement, operations without conversion, conversion of units, operations with conversion.
    f. Fractions (sixth grade level only)[2]: concepts of fractions, addition of fractions, subtraction of fractions, multiplication of fractions, and division of fractions.

These conclusions are limited to conditions similar to those involved in the present investigation: *i.e.*, to populations similar to those from which the subjects were selected; to aspects of mathematics achievement similar to those assessed; and to instructional programs at the fourth, fifth, and sixth grades similar to those used to teach mathematics to the populations sampled. In addition, the procedures used in the data analyses cannot be regarded as having yielded unique solutions.

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[1] Fourth grade pupils had not been instructed in all of the mathematics activities in geometry; therefore, only fifth and sixth grade data were examined.

[2] Since fourth and fifth grade pupils had not been instructed in all of the mathematics activities in fractions, only sixth grade data were analyzed.

CHAPTER IV

THE HORIZONTAL SEQUENCES

Research Objective and Procedures for Analyses

The second objective of the study was the following: to identify, at grade levels 4, 5, and 6, orthogonal and non-orthogonal horizontal sequences among several mathematics topics on the basis of the criterion, degree of relationship between adjacent topics.

Just as the six major topics have each been explored internally for evidence of increasing complexity of subtopics by means of simplex analysis, it had been originally intended to explore the same topics externally, each with the others, for evidence of some kind of substantive continuum or contiguity among them by means of circumplex analysis. Secondarily, it seemed desirable also to apply this latter procedure to the subtopics as a single set, ignoring the major categories of topics, for evidence of feasible horizontal sequencing cutting across the logical lines between the major topics.

Late in the project's timetable it was learned that Guttman (1965) had extended his radex model, of which both simplex and circumplex procedures are parts, so much so that he and competent peers were suggesting replacement of the earlier methods by his new "smallest space analysis" model. Accordingly, time was sought to permit exploration and application of smallest space analysis to our data.

Smallest Space Analysis of Topics

The new non-metric factorial approach was applied via its simplest computer program (SSA-I) (Lingoes, 1965b) to the correlation matrices of the major topics for grades 4, 5, and 6, respectively. Only five of the six major topics could be included in such analyses for grades 4 and 5 because student performance on Topic 6-Fractions was undistributed at these grade levels (See Appendix E). Consequently, only five variables were studied at grades 4 and 5, while these same five variables as well as the total six variables were subjected to separate analyses at grade 6.

In keeping with the approach taken in Chapter III, the topics--as defined by their test items--were reviewed to provide an a priori theoretical construct of similarities or contiguities of content and process among the topics. It immediately became apparent that Topic 2-Geometry differed most from the others in substantive content and in the emphasis placed on definition and classification as distinguished from problem-solving. Because each of the subtopics under Topic 5-Measurement included measures of length or distance in two of the four test exercises presented, it was assumed that this might have the closest affinity to Topic 2. On the other hand, Topic 5 shared with all the remaining topics a substantial emphasis on the solving of "word problems." Special content and emphasis was least evident in Topic 3-Relations. "New" content was notable in Topic 4-Numeration in the subtopics on sets and numeration bases. Topic 1-Number, operations, assumptions contained a heavy emphasis on pure algorithms. Topic 6-Fractions carried an almost exact parallel with Topic 1 in its relative emphasis on algorithms, "word problems," and concepts/terminology. To recapitulate and project a structure of relations among topics, a considerable commonality might be predicted to run through all topics because of the problem-solving emphasis. Beyond this Topic 2-Geometry might be predicted to stand to one side, Topic 4-Numeration might depart from commonality on another dimension of "new" emphasis, while Topic 1-Number, operations, assumptions and Topic 6-Fractions might be represented additionally on a dimension of "pure algorithm."

The technique of smallest space analysis yields a pattern of coordinates in k-dimensional space similar to factor analysis patterns. ($k < n$, the number of original variables; moreover, k tends to be approximately of the order of $\frac{n}{2}$.) The coordinates yield quantitative measures of distance in Euclidean space, but are not subject to interpretation as vectors with component factor loadings. The coordinate system defines relations in psychological space, but without commitment to an origin. For the simple cases treated here, five or six variables interpreted in two or three dimensions, the smallest space analysis and the principal axes factor analysis yield solutions that vary from highly similar to almost identical. (See Appendix F for factor solutions corresponding to the four smallest space solutions given in Tables 64 to 67.)

Fortunately, for simplicity of interpretation the smallest space analyses for five topics in grades 4, 5 and 6 and for all six topics in grade 6 are highly similar. Note that in Tables 64 to 67 and corresponding figures, Topics 3, 4, and 5 largely coincide with one another in the lower left corner of two-dimensional space, Topic 2 stands out to the right and Topic 1 is in the upper left sector. When Topic 6 makes its appearance in Table 67 and Figure 4, it finds its place close to Topic 5.

The results may be said to confirm the expectation that Topic 2 would stand apart from the others and that Topic 1 might also. Topic 5 bore no special relation to Topic 2 and the "newness" associated with Topic 4 was apparently no newer than other topics to these young students to whom perhaps it was all equally new. The "newness" is probably best put down as a distinctive characteristic only to those familiar with an older way of teaching and learning the mathematics of these grades. The single analysis involving Topic 6 gives no clear picture of the nature of that topic in that it does not follow Topic 1 but Topic 5. A review of the test exercises of Topic 5 reveals fractions, rational or decimal, are involved in perhaps 5 of the 16 exercises, hardly enough to explain the affinity shown. It seems simplest, hence best, to conclude that Topic 6 joins with Topics 3, 4 and 5 in reflecting the common problem-solving element, rather than any direct connection with the algorithmic emphasis in Topic 1.

These findings yield rather neat orthogonal sequences. With Topics 3, 4 and 5 so closely associated in all the analyses, the five-topic arrangements can be made in any order that puts Topics 1 and 2 in second and fourth positions, or vice versa, while Topics 3, 4 and 5 are distributed, as one prefers, among first, third, and fifth positions. With the advent of Topic 6 in the grade 6 analysis, it becomes desirable to try to find a place for it that will maximize its deviation from one of Topics 3, 4 and 5 at either end of the topical sequence. By computing the several sequences generally specified above, and paying chief attention to Table 67 and its corresponding figure because the closely associated Topics 3, 4 and 5 are most widely separated there, a

Table 64

Guttman-Lingoes Smallest Space Coordinates for Grade 4 (N = 244)

Topic	Dimension 1	Dimension 2
1	-818	1375
2	1779	346
3	-110	-722
4	-487	-583
5	-363	-416

Table 65

Guttman-Lingoes Smallest Space Coordinates for Grade 5 (N = 261)

Topic	Dimension 1	Dimension 2
1	-235	1401
2	1843	91
3	-553	-455
4	-497	-518
5	-558	-519

Table 66

Guttman-Lingoes Smallest Space Coordinates for Grade 6 (N = 240)

Topic	Dimension 1	Dimension 2
1	- 3	1826
2	1368	-164
3	-184	-663
4	-923	-713
5	-258	-286

Table 67

Guttman-Lingoes Smallest Space Coordinates for Six Topics in Grade 6

Topic	Dimension 1	Dimension 2
1	-607	1710
2	1403	91
3	485	-1063
4	-954	-1156
5	-196	179
6	-131	239

Figure 1

Smallest Space Coordinates for
Grade 4 (N = 244)

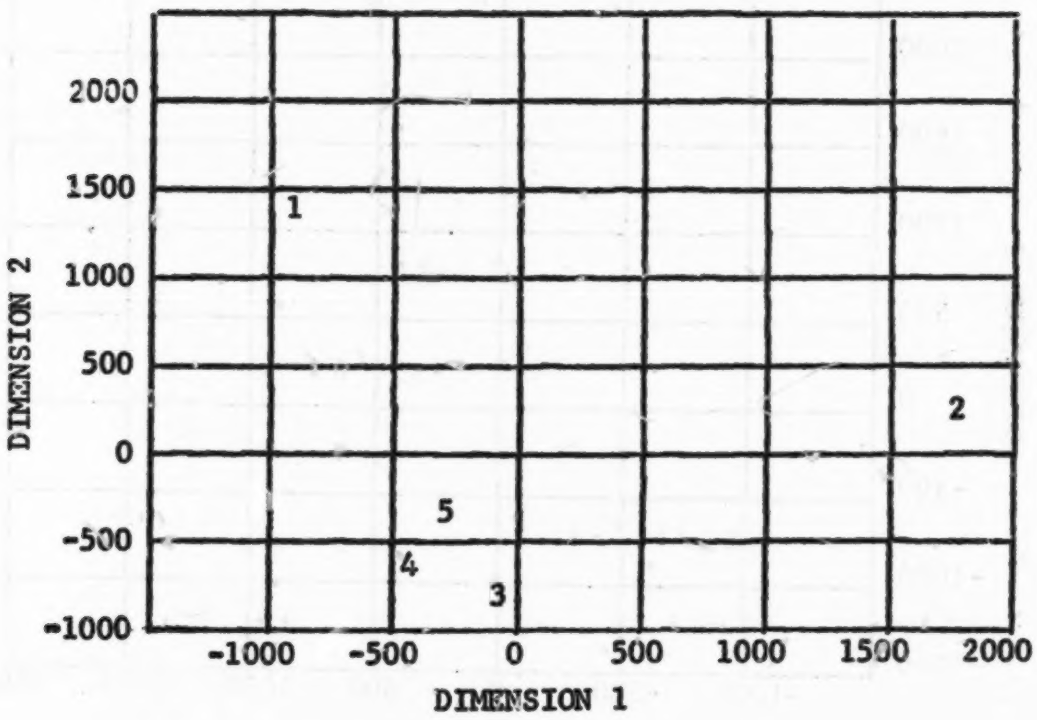


Figure 2

Smallest Space Coordinates for
Grade 5 (N = 261)

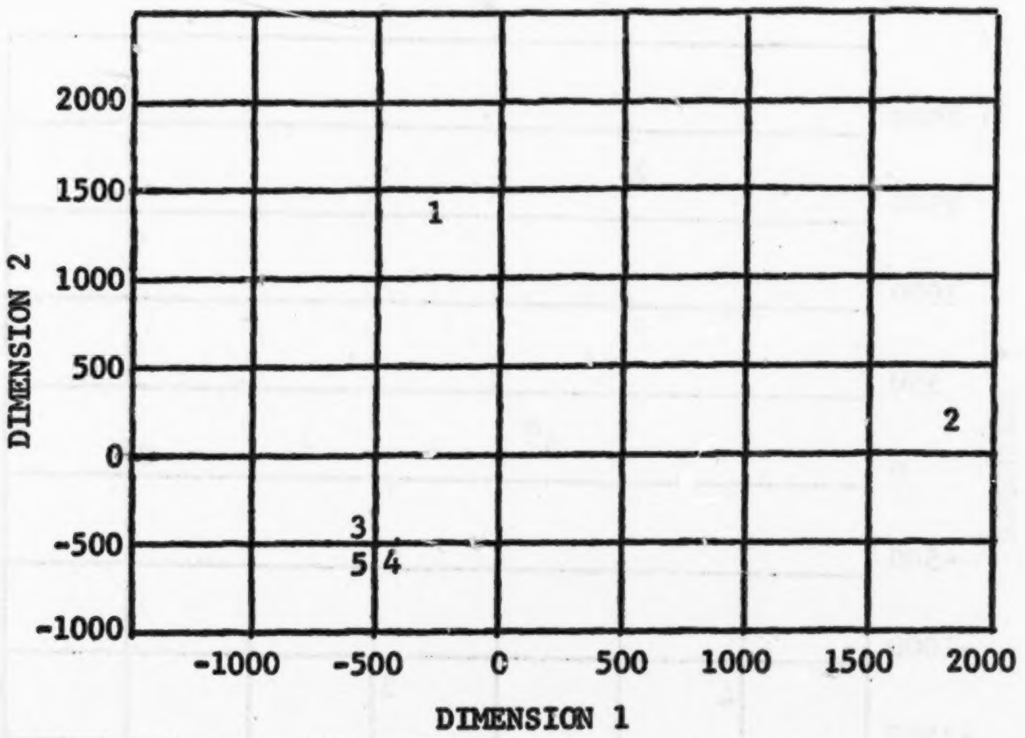


Figure 3

Smallest Space Coordinates for
Grade 6 (N = 240)

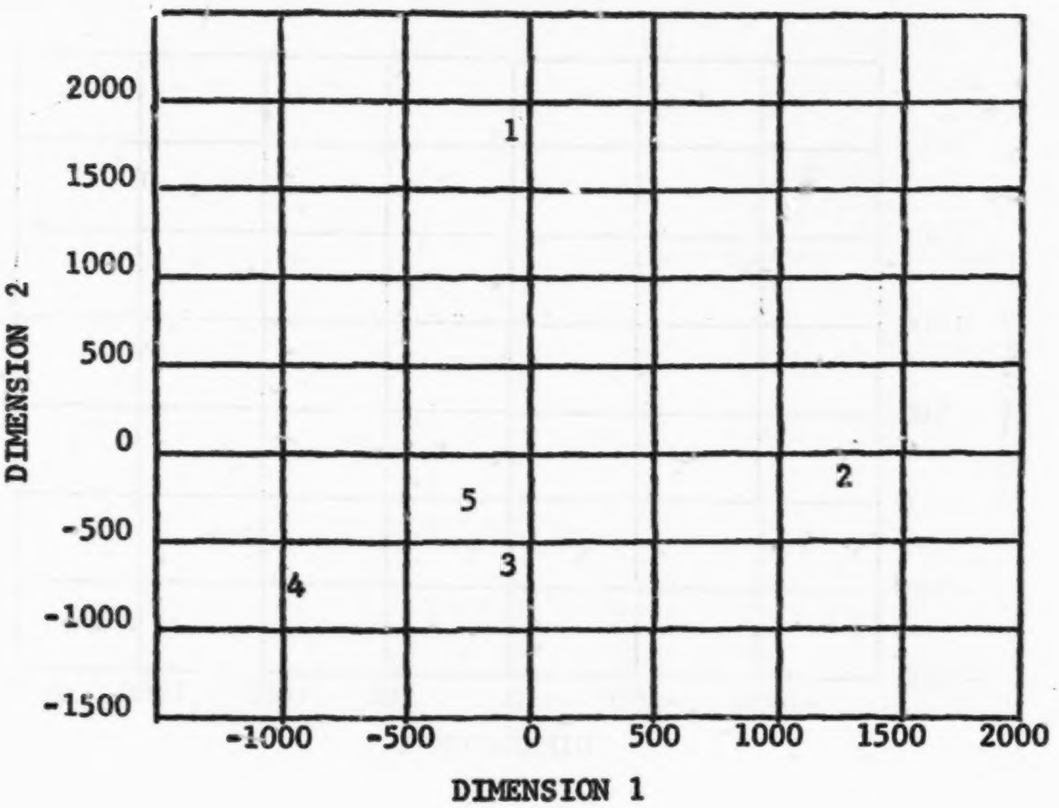
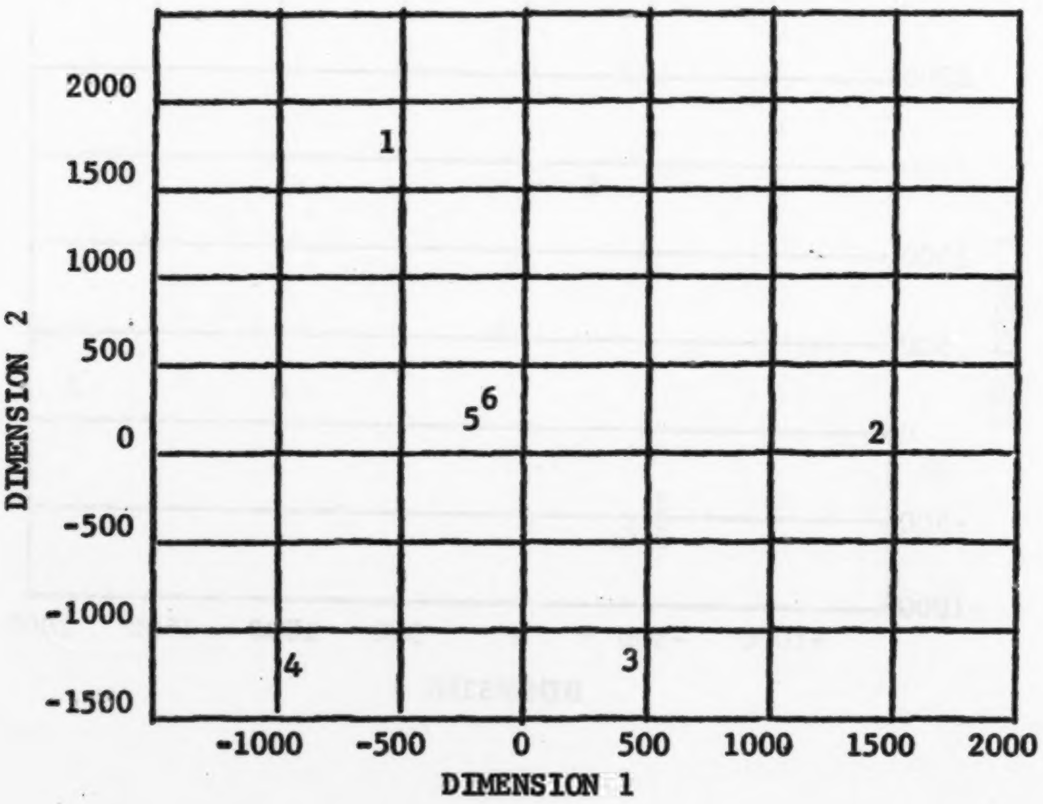


Figure 4

Smallest Space Coordinates for Six Topics
in Grade 6 (N = 240)



"generally best" sequence would appear to be Topics 5, 2, 4, 1, 3, 6, or the reverse. This empirical solution does violence only to the a priori assumption that Topic 5-Measurement should show some affinity for Topic 2-Geometry. If that rational criterion is to be respected and the 6-topic pattern to be given less special attention, a "generally best" sequence would be Topics 5, 1, 4, 2, 3, 6, or the reverse.

By the same token, these data lend themselves to a rather straightforward non-orthogonal solution of Topics 2, 3, 4, 5, 6 and 1, or the reverse. This sequence gives a best fit to the fourth grade data and both analyses of the sixth grade data while the fifth grade data are fitted by any sequence that places Topics 1 and 2 at opposite extremes.

Smallest Space Analysis of Subtopics

Turning next to the subtopics as a single set, the Smallest Space Analysis III computer program (Lingoes, 1965d) was used to analyze the more complex dimensional structure to be expected of the 20 subtopics in Topics 1-5 and the added 5 subtopics of Topic 6. Since these analyses yielded four to six interpretable dimensions, it would be impractical to attempt an explanation in graphical terms, taking the dimensions pair by pair for each grade group. Instead only the tables for the smallest space analyses for the three grade groups for the 20 subtopics of Topics 1-5 and the analysis of the total 25 subtopics for grade 6 are presented for discussion. Taking our cue not only from the original constructs for the six major topics, but also from the smallest space analysis of those topics, we might anticipate separation of the subtopics of Topic 1-Number, operations, assumptions and of Topic 2-Geometry to cluster by themselves, while a general factor common to all topics, but especially Topics 3-6, might be expected to appear. Tables 68-71 are for the same grade groups and topics as Tables 64-67.

As pointed out earlier, the dimensions resulting from a smallest space analysis must not be confused with factors. By the same token, the coefficients for each subtopic must not be considered factor loadings on those dimensions to be explained singly. Rather each subtopic in Table 68 must be considered a point in 6-space related to all the others by the pattern of its six coefficients. It should be noted, however, that the normalized varimax rotation has the effect of maximizing the coefficients in such a way as to give each subtopic one or two large coefficients while at the same time restricting the number

Table 68

Smallest Space Analysis III for Subtopics in Grade 4 (N = 244)

Subtopics	Dimensions 1	2	3	4	5	6
1	.03	.11	.09	-.02	.48**	.04
2	.18	.41*	.48**	-.09	.27	.13
3	.05	.36*	.63**	.10	.02	.09
4	.07	-.03	.68**	.15	.12	-.05
5	.28	.52*	.19	.10	.06	.11
6	.04	.06	.03	.32*	-.08	.07
7	.14	.00	.12	.49**	.35*	.11
8	.23	.35*	.19	.51**	.00	-.05
9	.28	.56**	.09	.35*	.05	.18
10	.43*	.11	-.05	.26	.13	-.07
11	.19	.27	.03	.17	.11	.70**
12	.44*	-.05	.16	.27	.25	.24
13	.27	.55**	.06	.11	.37*	.15
14	.57**	.21	-.06	-.11	.31	.03
15	.46**	.26	.21	.18	.29	.22
16	.58**	.22	.27	.01	-.09	.14
17	.55**	.19	-.04	.23	.07	.09
18	.46**	.32*	.33*	.11	.02	.15
19	.41*	.44**	.29	.12	.19	.17
20	.52**	.18	.25	.18	-.19	.09
Sum of Squares	2.60	1.95	1.62	1.12	0.94	0.80

** Substantial coefficients

* Appreciable coefficients

Table 69

Smallest Space Analysis III for Subtopics in Grade 5 (N = 261)

	Dimensions				
Subtopics	1	2	3	4	5
1	.16	.05	.14	.12	.15
2	.60**	-.01	.21	.16	.24
3	.50**	.16	.29	.25	.20
4	.53**	.31	.09	.12	.19
5	.18	.10	.51**	.31	.17
6	.10	.10	.11	.10	.67**
7	.33*	.23	.38*	.16	.31
8	.34*	.52**	.04	.18	.32*
9	.47**	.13	.48**	.02	.23
10	.45**	.34*	.13	.39*	.07
11	.13	.11	.61**	.14	.03
12	.42*	.29	.21	.41*	-.09
13	.09	.05	.20	.59**	.16
14	.20	.81**	.31	.12	.07
15	.54**	.19	.33*	.16	.04
16	.51**	.32*	.11	.23	.15
17	.39*	.21	.15	.48**	.14
18	.48**	.34*	.08	.42*	.08
19	.60**	.11	.35*	.10	.01
20	.53**	.29	.08	.21	.08
Sum of Squares	3.42	1.75	1.66	1.53	0.99

**Substantial coefficients

*Appreciable coefficients

Table 70

Smallest Space Analysis III for 20 Subtopics in Grade 6 (N = 240)

Subtopics	Dimensions 1	 2	 3	 4
1	.02	.15	.00	.55**
2	.19	.68**	.09	.09
3	.33*	.48**	.25	.13
4	.40*	.34*	.13	.31
5	.21	.28	.36*	.08
6	.11	.11	.59**	-.13
7	.22	.46**	.52**	.22
8	.58**	.42*	.30	-.06
9	.33*	.33*	.36*	.09
10	.54**	.30	.40*	-.16
11	.49**	.07	.09	.03
12	.57**	.30	.20	.12
13	.25	-.10	.42*	.24
14	.46**	.12	.25	.07
15	.48**	.44*	.22	-.13
16	.62**	.11	.14	.09
17	.60**	.36*	.18	.05
18	.59**	.33*	.20	.10
19	.27	.65**	.05	.22
20	.72**	.34*	.14	.06
Sum of Squares	3.92	2.61	1.67	0.71

**Substantial coefficients

*Appreciable coefficients

Table 71

Smallest Space Analysis III for 25 Subtopics in Grade 6 (N = 240)

	Dimensions				
Subtopics	**1**	**2**	**3**	**4**	**5**
1	-.04	.15	.03	.64**	.02
2	.17	.66**	.15	.06	.03
3	.33*	.52**	.27	.16	.09
4	.41*	.33*	.14	.33*	.13
5	.17	.25	.42*	.06	.03
6	.13	.04	.47**	.00	.25
7	.21	.44*	.53**	.18	.08
8	.58**	.31	.44*	.04	-.02
9	.28	.28	.52**	.00	.13
10	.54**	.21	.45**	-.07	.10
11	.43*	.08	.21	-.03	-.03
12	.54**	.30	.22	.13	.10
13	.13	.05	.19	.04	.72**
14	.50**	-.01	.31	.20	.07
15	.50**	.36*	.29	-.08	-.01
16	.58**	.22	.07	.02	.17
17	.59**	.36*	.19	-.05	.18
18	.60**	.36*	.19	.08	.15
19	.25	.70**	.13	.16	-.04
20	.68**	.38*	.18	-.01	.08
21	.32*	.49**	.45**	.13	.16
22	.47**	.49**	.14	.20	.23
23	.52**	.44*	.19	.23	.28
24	.59**	.21	.33*	.36*	.07
25	.35*	.01	.41*	.29	.00
Sum of Squares	4.70	3.19	2.42	1.01	0.90

** Substantial coefficients

* Appreciable coefficients

of subtopics having a large coefficient on any one dimension. Two consequences are that (1) any dimension with a large number of subtopics having large coefficients suggests the possibility of an irreducible common element in the several subtopics, and (2) large coefficients may be used as aids in detecting common patterns of dimensionality among subtopics. To aid in applying both of these lines of interpretation, each coefficient of .45 or higher has been marked with a double asterisk and each other coefficient of .32 or higher has been marked with a single asterisk. The reader may prefer to ignore them and make his own judgments on the basis of the relative magnitudes of the coefficients. The critical values .45 and .32 have been chosen because in factor analysis, where the coefficients represent coordinates of orthogonal dimensions from a presumably significant zero-point or origin, the squares of the coefficients measure the proportion of the total variance of each variable accounted for by that factor, and the squares of .45 and .32 are .20 and .10, respectively. Also, the horizontal lines in Table 68 separate the subtopics by major topic.

Table 68, then, reflects a considerable irreducible common element largely defined by Dimension 1 as being substantially represented in the subtopics of Topics 3, 4 and 5, a finding previously indicated by the analysis of the major topics. Beyond this the subtopics of Topic 1 are defined largely by large coefficients on Dimensions 2 and 3, and those of Topic 2 by large coefficients on Dimension 4. Topics 3, 4 and 5 are differentiated secondarily by the fact that the subtopics of Topic 5 have some large coefficients on Dimension 2, those of Topic 4 also appear on Dimension 5, while those of Topic 3 show no clear pattern of high coefficients on any other dimension. Dimension 6 has the sole virtue of helping define Subtopic 11 as rather unrelated to any of the others. This subtopic involves interpretation of a chart, which might be considered a somewhat discrete skill or at least one not taught integrally with other measured skills at this grade level.

Subtopic 1-Addition of whole numbers, as represented by the four test exercises included, appears so thoroughly mastered even at grade 4 as to be undistributed except for careless errors and hence little related to any other subtopic, including the other subtopics under

Topic 1. Its definition by Dimension 5 is fortuitous. If a seventh or eighth dimension had been added, it might well have been found there, mildly associated with Subtopic 2-Subtraction of Whole Numbers. Otherwise, Dimension 5 serves to reflect the "newness" element undetected in the previous analyses of major topics. Subtopics 13, 14 and 15, having to do with sets, bases other than ten, and the placeholding value of zero, respectively, are moderately represented on this dimension.

Dimensions 2 and 3 serve to differentiate two aspects of mastering operations with whole numbers. Dimension 3 points more toward routinizing mastery of the algorithm, with subtraction, multiplication and division having successively higher coefficients. Dimension 2 relates the better mastered algorithms to generalized concepts about them.

Much of the above interpretation of Table 68 may be made similarly for Table 69. Dimension 1 reflects an even more substantial common element in the data of grade 5 than was found in grade 4. Here , 10 or half the coefficients are substantial and 4 more are appreciable. Each of the remaining six subtopics are somewhat unique, while all the problem-solving subtopics are heavily represented. Moreover, each of the major topics is represented on Dimension 1 by at least two subtopics with appreciable coefficients. The five major topics are represented by appreciably high coefficients as follows: Topic 1 on Dimension 1 only; Topic 2 on Dimensions 1 and 5; Topic 3 on Dimensions 1, 3 and 4; Topic 4 on Dimensions 1, 2 and 3; and Topic 5 on Dimensions 1 and 4. Topic 2 is most readily identified by Dimension 5, where all three subtopics have appreciable or substantial coefficients.

Subtopic 1-Addition of whole numbers has no coefficient larger than .16, indicating even more clearly than at grade 4 that it is undistributed, except for careless errors, because of mastery. The "newness" element in Subtopics 13, 14 and 15 is overshadowed by the more substantial association of each of these subtopics with subtopics of other major topics. Subtopic 11 is now integrally related in pattern to several other subtopics (5, 7, 9) given direct emphasis at this grade level. Subtopics 2, 3 and 4 have come to be most related to the general problem-solving element at this grade level.

Table 70 again shows a substantial element in Dimension 1. Ten of the coefficients are substantial and 3 others are appreciable.

Again it seems a general problem-solving element, all subtopics containing such an element being represented by substantial or appreciable coefficients. The five major topics are defined by appreciably high coefficients as follows: Topic 1 on Dimensions 1, 2 and 4; Topic 2 on Dimensions 2 and 3; Topic 3 on Dimensions 1, 2 and 3; Topic 4 on Dimension 1 only; and Topic 5 on Dimensions 1 and 2. Topic 2 continues a relatively unique element except for Subtopic 8, which is definitely problem-solving in geometry. Subtopic 1 is still uniquely undistributed, its appearance on Dimension 4 being unaccompanied by other large coefficients on that dimension. Subtopic 2-Subtraction of whole numbers might appear to have joined Subtopic 1 as unrelated to the major dimension of problem-solving because of its low coefficient on Dimension 1, but the broad extent of large coefficients on Dimension 2 suggests that the dimensionality of problem-solving must be thought of here as a pattern of these two statistical dimensions, and Subtopic 2 has a large coefficient on Dimension 2.

The "newness" element remarked at grade 4 is again missing at grade 6. Also, Subtopic 11 appears now to be thoroughly a part of the general problem-solving element reflected in Dimensions 1 and 2.

Table 71 departs very little from Table 70, as may be ascertained by casual inspection. The chief effect of including the 5 subtopics of Topic 6-Fractions is to show that these subtopics are reflected in the problem-solving element of Dimensions 1 and 2. The addition of Dimension 5 demonstrates the singular dimensionality of Subtopic 13-Sets. Dimension 3 now takes on the aspect of defining the special relationships among the Geometry subtopics.

The analysis of subtopics does nothing to call into question the orthogonal and non-orthogonal sequences proposed at the end of the analysis of major topics. In view of the fact that "problem-solving as chiefly represented in "word problems," appears to be the chief element among the major topics and is represented carefully in every topic and even in the subtopics of Topics 1 and 6, it could be argued that separation of "word problems" from the other subtopics might make for better orthogonal arrangements. Similarly, the content of Subtopic 11-Maps, charts and graphs could be expanded to make a separate orthogonal unit. However, once the present units are broken we are forced to redefine the problem of achieving orthogonality or non-orthogonality of sequences. Topics have to be, or be deemed to be, relatively homogeneous elements in order to talk about orthogonality (or non-orthogonality) of topic sequences. Insofar as there seems to be good reason to teach each topic and its applications (problem-solving) together, there is no profitable use of the potential orthogonality of "word problems" except to view them as mildly orthogonal variations within non-orthogonal topics. No other consistently orthogonal or non-orthogonal relations among subtopics appear to run through the data of this study. Even Subtopic 11, which appears orthogonal at grade 4, becomes progressively more non-orthogonal in successive grades.

Implications of Analyses

The chief implication of the analyses for horizontal sequencing of topics is that the considerable commonality of problem-solving means that there is no indicated proper or "best" sequence, either orthogonal or non-orthogonal. So far as operations with whole numbers are concerned, addition is considerably mastered by grade 4, subtraction and multiplication seem in process of being mastered, division is being taught directly and gradually mastered. Insofar as division is being taught, subtraction and multiplication are being reviewed and practiced in the process. Geometry stands out as unique from other topics so might be taught as a unit at the beginning or end of each year. Beginning of the year suggests itself because of the use of spatial visualization in summer activities. With problem-solving an integral emphasis in all instruction, it would appear that the special topics might be taught as at present, each topic being built up along the lines of increasing complexity as indicated by the simplex analysis. Individualization of instruction by programing suggests itself since the simplex analysis indicates appropriate sequences within topics. As between topics the course is generally clear and open. An annual cycle of topics, perhaps starting with geometry, incidental review of computational processes, and introduction of topics by assessing the current mastery level of subtopics along the lines of the simplex seem the next steps. Tests based on the simplex analysis should help greatly in planning and conducting group and individual instruction. If the cycle is followed, longer pretests than those in the present battery can be used with profit at the beginning of each unit. Hopefully, as time goes on, more will be learned earlier in many if not all the topics. The simplex sequences seem the chief key to guiding lower grade placement of opportunities to learn in each topic.

Summary of Conclusions

Several conclusions were supported by the analyses of the horizontal sequences and are listed below.

1. The only topics that are set off from the others and from each other are geometry and operations with whole numbers. Relations, numeration, and measurement show great commonality at all grade levels and are joined at grade 6 by operations with fractions. The commonality is generally attributed to the "problem-solving" element represented in every topic.

2. Analysis of the subtopics as a single set generally confirmed the above conclusions reached by analysis of the broader topics. The great commonality, despite the fundamental statistical properties of varimax rotation, gives unique support to the evidence from analysis by whole topics.

3. The best empirical orthogonal sequence is measurement, geometry, numeration, whole numbers, relations, and fractions, or the reverse. If this sequence is dismissed because of the rational association of measurement with geometry, the next best orthogonal sequence is measurement, whole numbers, numeration, geometry, fractions, and relations, or the reverse.

4. The best non-orthogonal sequence is geometry, relations, numeration, measurement, fractions, and whole numbers, or the reverse.

5. In view of the great commonality due to "problem-solving", it was concluded that the best teaching sequence would be to teach geometry first, review operations with whole numbers incidentally, but use the simplex sequences within each of the other topics to achieve continuity and modest challenge.

CHAPTER V

SUMMARY

Problem

Purpose and Objectives

This study's purpose was to use procedures based on Guttman's (1954, 1965) radex theory in identifying, empirically, vertical and horizontal sequences for organizing mathematics content at grade levels 4, 5, and 6. The specific objectives were as follows:

1. to identify, at grade levels 4, 5, and 6, vertical sequences among subtopics within several mathematics topics on the basis of the criterion, level of complexity of the subtopics;
2. to identify, at grade levels 4, 5, and 6, orthogonal and non-orthogonal horizontal sequences among several mathematics topics on the basis of the criterion, degree of relationship between adjacent topics.

Educational Significance

Information yielded by this and similar studies could be useful in establishing instructional sequences for teaching mathematics. That is, more effective sequences can be established when the structure of the interrelations within the material to be taught is considered jointly with such planning factors as the characteristics of the pupils and the practical exigencies of the situation. This planning process can be strengthened by identifying sequential arrangements among mathematics activities on the empirical basis of the achievement of pupils who have been taught in a given mathematics instructional program.

Related Literature

The present investigators located no other projects which were specifically designed to use Guttman's or other scaling procedures as adjuncts in establishing sequences among activities in contemporary mathematics instructional programs for elementary school pupils.

Guttman (1954) and Kaiser (1962) cited a simplical arrangement among six numerical tasks culled from Thurstone's (1938) work as illustrations in their discussions of procedures for simplex analyses. More tangential to the present project, Gabriel (1954), Guttman (1954, 1957, 1965), Humphreys (1960), and Lingoes (1965a) demonstrated that procedures related to Guttman's radex theory yielded satisfactory descriptions of order relations among various types of psychological data.

Procedure

Research Design

The plan for collecting data pertinent to the project's purpose involved administering to pupils a test designed to assess achievement of mathematics instructional objectives at grade levels 4, 5, and 6.

Subjects

Eligible for inclusion in the samples were those pupils in the Clarke County (Georgia) School District who were taught in the mathematics instructional program at grade levels 4, 5, and 6 during the 1963-64 academic year and who satisfied certain other selection criteria. Pupils were selected at each grade level during the spring of 1964. The total group included 765 subjects: 244 in the fourth grade sample, 261 in the fifth grade sample, and 260 in the sixth grade sample.

The Mathematics Achievement Test

The mathematics data were interval scores on a test designed to assess achievement of instructional objectives for the following selected topics and subtopics.

1. Number, operations, and assumptions: addition, subtraction, multiplication, division, and laws and generalizations.
2. Geometry: definitions of geometric terms, recognition of geometric figures, and measurement of geometric figures.
3. Relations: equality; order relations; maps, graphs, charts; and functions.
4. Numeration: sets; other number bases; reading, writing, using numbers; and Roman numerals.
5. Measurement: concepts of measurement, conversion of units, operations without conversion, and operations with conversion.

6. Fractions: concepts of fractions, addition of fractions, subtraction of fractions, multiplication of fractions, and division of fractions.

General Procedures

Major steps involved in data collection included the following activities. Classes of pupils were selected from the total number available on the basis of criteria related to the teachers' training and experience; then, pupils who were eligible in terms of the remaining subject selection criteria were identified. Classroom teachers' administered the test. Prior to the test administration, a detailed testing schedule was formulated with the cooperation of appropriate school personnel. The testing sessions were scheduled so that all selected classes were administered the tests at approximately the same time on May 8, 1964. Meetings were held with teachers and principals to orient them to the project. Following the orientation sessions, conferences were arranged for individual teachers who wished additional assistance in preparing for the test administration.

The Vertical Sequences

Essentially, identifying the vertical sequences involved using simplex analyses to examine the structure of intercorrelations among pupils' mathematics subtopic scores. That is, the investigators' task was to determine whether the simplex model fits data ordered a priori on the basis of relative complexity.

The criterion for complexity was degree of inclusiveness. In the context of the present study, degree of inclusiveness was specified as the extent to which any one subtopic within a set of subtopics involves activities characteristic of other subtopics in that set plus additional activities.

The first step in the analyses was to obtain the intercorrelation matrix for the set of subtopics encompassed by each one of the six topics, viz.: number, operations, assumptions; geometry; relations; numeration; measurement; and fractions. Examining the structure of these intercorrelations involved three major steps: ordering the subtopics; determining the relative complexity of the subtopics; and assessing the goodness of fit of the simplex model to the empirical data. Following Guttman's (1954, 1965) approach, the subtopics within an intercorrelation matrix were ordered on an a priori basis. Procedures proposed by Kaiser (1962) were used to determine the relative complexity of the subtopics and the goodness of fit of the model to the data.

Briefly, for all sets of subtopics at the three grade levels, the results of the analyses indicated the following. The simplex model did fit satisfactorily data ordered a priori on the basis of relative complexity. The complexity loadings increased across subtopics arranged a priori to vary in complexity from simple to complex. The index q^2 and the degree of correspondence between the observed and reproduced R_{jk} and r_{jk} matrices, respectively, were satisfactorily high.

The Horizontal Sequences

Horizontal sequences were explored by means of the Guttman-Lingoes (1965a,d) smallest space analysis. This non-metric factorial approach permitted analysis of the closeness of relations among the six major topics within a two-dimensional Euclidean space. The test exercises under the various topics were first inspected for rational a priori judgments of overlap. Then the topics were graphically represented in two-dimensional space for each grade group separately. Next, confirmations and disconfirmations of a priori expectations were noted. Finally, two solutions were presented, orthogonal and non-orthogonal, i.e., with consecutive topics least closely related and most closely related, respectively. The same sequence of steps was followed in analyzing relations among subtopics as a single set at each grade level.

Conclusions and Limitations

Conclusions Relevant
to the Vertical Sequences

Several conclusions were supported by the analyses of the vertical sequences. These conclusions are the following.

1. At grade levels 4, 5, and 6, a vertical hierarchy along the complexity dimension is present among subtopics within the topics analyzed.

2. Within the respective topics, the hierarchical orders among subtopics from least to most complex are the following.

    a. Number, operations, and assumptions: addition, subtraction, multiplication, division, and laws and generalizations.

b. Geometry (fifth and sixth grade levels only)[1]: definitions of geometric terms, recognition of geometric figures, and measurement of geometric figures.

c. Relations: maps, graphs, and charts; equality; functions; and order relations.

d. Numeration: Roman numerals; reading, writing, and using whole numbers and decimals; other numeration bases; and sets.

e. Measurement: concepts of measurement, operations without conversion, conversion of units, operations with conversion.

f. Fractions (sixth grade level only)[2]: concepts of fractions, addition of fractions, subtraction of fractions, multiplication of fractions, and division of fractions.

Conclusions Relevant to the Horizontal Sequences

Several conclusions were supported by the analyses of the horizontal sequences and are listed below.

1. The only topics that are set off from the others and from each other are geometry and operations with whole numbers. Relations, numeration, and measurement show great commonality at all grade levels and are joined at grade 6 by operations with fractions. The commonality is generally attributable to the "problem-solving" element represented in every topic.
2. Analysis of the subtopics as a single set generally confirmed the above conclusions reached by analysis of the broader topics. The great commonality, despite the fundamental statistical properties of varimax rotation, gives unique support to the evidence from analysis by whole topics.
3. The best empirical orthogonal sequence is measurement, geometry, numeration, whole numbers, relations, and fractions, or the reverse. If this sequence is dismissed because of the rational association of measurement with geometry, the next best orthogonal sequence is measurement, whole numbers, numeration, geometry, fractions, and relations, or the reverse.
4. The best non-orthogonal sequence is geometry, relations, numeration, measurement, fractions, and whole numbers, or the reverse.

[1,2] Fourth grade pupils had not been instructed in all of the mathematics activities involved in geometry; similarly, both fourth and fifth grade pupils had not been taught all of the activities involved in fractions. Consequently, the analyses were restricted, as indicated, to pupils who had received instruction in all activities involved in all subtopics.

5. In view of the great commonality due to "problem-solving", it was concluded that the best teaching sequence would be to teach geometry first, review operations with whole numbers incidentally, but use the simplex sequences within each of the other topics to achieve continuity and modest challenge.

Limitations

Inherent in the procedures and samples used in this study are at least six limitations to internal and external validity which merit consideration. Extension of conclusions beyond these limitations will require further investigations.

First, not all aspects of mathematics achievement were sampled. The domains of relevant behavior are extensive and time and funds were restricted. Further research will be required to study behavior not assessed in the present study.

Second, generalizations are limited, to some extent, to instructional programs of the type used to teach the pupils mathematics and to populations of pupils similar to those at grade levels 4, 5, and 6 in the Clarke County (Georgia) School District. Since mathematics primarily represents specifically taught behavior, such limitations would seem to apply to any single study of mathematics achievement which is not extremely extensive in scope--more extensive than the scope possible within the budgetary and time restrictions of the present study. The mathematics instructional program in Clarke County has a number of positive features and for several years has been the focus of intensive efforts for improvement. Also, efforts have been made to attain consistency among classes in procedures related to the several elements of the mathematics program such as scope, sequence, timing, methods, materials, allocation of personnel and facilities, and organization of pupils for instruction. In addition, none of the available evidence indicated that the pupils employed in the samples represented a markedly atypical population. However, generalizations are limited to populations and instructional programs similar to the ones described herein. Further work will be necessary to extend these generalizations to different populations which have other backgrounds of mathematics training.

Third, because of a rather limited amount of testing time in the schools, it was necessary to restrict the number of test items which could be used for assessing achievement in the mathematics subtopics.

As a result, the reliability coefficients for some of the subtopic scores were relatively low. However, they were considered sufficiently high to yield, for research purposes, some preliminary information. It would be helpful to replicate the study using subtopic tests of greater length and higher reliability.

Fourth, because of time and budgetary restrictions as well as other practical considerations, the focus of the study was limited to the content of the mathematics instructional program at grade levels 4, 5, and 6. Similarly important is an examination of such problems as the following: the nature of the relationships proposed for study herein at other grade levels, developmental trends in such relationships across grade levels from the primary school through the high school, and so on. Such extensions will require further investigations which encompass the content of the instructional program at other grade levels.

Fifth, any analysis of sequences is dependent on the manner in which subtopics are conceived and combined into topics. For example, if the "word problems" included in Subtopics 1, 2, 3, and 4 had been separately grouped as a subtopic of "word problems" under Topic 1, a different simplex problem might have presented itself. Similarly, if the "word problem" subtopics under the several major topics had been grouped to form another major topic of "word problems", the smallest space analyses might have differed. This study, then, is of sequences of the topics and subtopics as organized and generalizations must be limited to this or highly similar organizations of topics and subtopics.

Sixth, conclusions regarding the vertical hierarchical orders among subtopics are limited to describing the sufficiency of the fit of the _a priori_ judgments of hierarchical order from least to most complex. No unique empirical solution to the vertical sequence is achieved, or even attempted, as might be done in this case for the small number of alternative sequences possible.

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APPENDICES

APPENDIX A

COOPERATING SCHOOL PERSONNEL

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Administrative Personnel

Superintendent: Mr. Sam W. Wood; Associate Superintendent: Mr. Alton A. Ellis; Guidance and Testing Director: Mr. J. C. Mullis.

Principals

Miss Allea Betts, Miss Dorothy Firor, Mr. Robert Garrard, Miss Grace Hancock, Mr. Don Hight, Mrs. Rosa Tarpley, Miss Helen Treanor, and Miss Annie Wallace.

Teachers

Mrs. Vera Bennett, Mrs. Julia Brackett, Mrs. Marjorie Brightwell, Mrs. Eva Brown, Mrs. Marlene Bush, Mrs. Margaret Callahan, Mrs. Eugenia Coggin, Mrs. Beth Cooper, Mrs. Carolyn Davis, Mrs. Hazel Davis, Mrs. Clara Doster, Mrs. Louise Eidson, Miss Frances Goodwin, Miss Mildred Johnson, Miss Vada Kent, Mrs. Mary McCutchen, Mrs. Doris Madden, Mrs. Gertrude Martin, Mrs. Editha Mills, Mrs. Florrie Oldham, Mrs. Marguerite Patat, Mrs. Lou Plant, Mrs. Diane Shortall, Mrs. Lily Tabor, and Miss Annette Zuber.

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APPENDIX B

INSTRUCTIONAL OBJECTIVES AND SELECTED TEST ITEMS

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Explanatory Note

This appendix consists of instructional objectives and test items classified by mathematics topics and subtopics. For each subtopic, the instructional objectives are presented first. In this presentation, the symbol X is used to indicate that an instructional objective is pertinent at a given grade level. When an instructional objective is not pertinent at a given grade level, the abbreviation NA is used to indicate "not applicable". After the instructional objectives are presented, the test items selected to sample achievement of these objectives are listed.

Topic: Number, operation, assumptions

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Subtopic: Addition

Instructional Objectives	Grade Level 4	5	6
1. To add 2, 3, and 4 place numbers	X	X	X
2. To add columns of 1, 2, 3, and 4 place numbers	X	X	X

```
  29
+ 17
```

F 12
G 36
H 43
J 56
K None of these

```
  28
  59
  94
+ 96
```

A 256
B 257
C 267
D 277
E None of these

```
 1111
+ 999
```

A 2000
B 2010
C 2100
D 2110
E None of these

Joan picked 23 tomatoes from her garden on Monday, 36 on Tuesday, and 18 on Wednesday. How many tomatoes did she pick in all during the three days?

F 67
G 77
H 86
J 87
K 96

Topic: Number, operation, assumptions

Subtopic: Subtraction

Instructional Objectives	Grade Level 4	5	6
1. To subtract 2, 3, and 4 place numbers with regrouping	X	X	X

$$\begin{array}{r} 6007 \\ -3928 \\ \hline \end{array}$$

A 179
B 1079
C 2079
D 2179
E None of these

$$\begin{array}{r} 17 \\ -\ 9 \\ \hline \end{array}$$

A 6
B 8
C 9
D 12
E None of these

$$\begin{array}{r} 821 \\ -514 \\ \hline \end{array}$$

A 207
B 307
C 310
D 317
E None of these

Bill weighs 109 pounds.
George weighs 127 pounds.
How many pounds difference
is there in their weights?

F 18
G 20
H 28
J 118
K 136

Topic: Number, operations, assumptions

Subtopic: Multiplication

Instructional Objectives	Grade Level 4	5	6
1. To multiply using 1, 2, and 3 place multipliers, including zeros and regrouping.	X	X	X

800
x 60

F 480
G 4,800
H 48,000
J 480,000
K None of these

184
x 25

F 1,288
G 3,772
H 4,600
J 37,720
K None of these

580
x 304

A 4,060
B 19,720
C 176,020
D 176,320
E None of these

The school bus makes three round trips each day. How many round trips does the bus make in 260 days?

A 263
B 520
C 680
D 780
E 980

Topic: Number, operations, assumptions

Subtopic: Division

Instructional Objectives	Grade Level 4	Grade Level 5	Grade Level 6
1. To divide by 1 and 2 digit numbers with and without remainders	X	X	X

$5\overline{)355}$

F 61
G 70
H 71
J 710
K None of these

$3\overline{)1821}$

A 60
B 67
C 670
D 1607
E None of these

$89\overline{)8366}$

A 9.4
B 94
C 904
D 940
E None of these

$32\overline{)165}$

F 5

G $5\frac{5}{32}$

H $5\frac{15}{32}$

J $6\frac{5}{32}$

K $6\frac{15}{32}$

Topic: Number, operations, assumptions

Subtopic: Laws and generalizations

	Grade Level		
Instructional Objectives	4	5	6
1. To demonstrate understanding of generalizations concerning 1 and 0	X	X	X
2. To demonstrate understanding of addition and subtraction as inverse processes	X	X	X
3. To demonstrate understanding of the commutative and associative laws	X	X	X

$6 \times \square = 0$

Which one of the numerals goes in the box to make the sentence true?

- A 0
- B 1
- C 6
- D 10
- E $\frac{1}{6}$

Which sentence expresses the property of associativity in addition?

- A $rs = s + r$
- B $r(s+t) = (r+s)t$
- C $r + s = s + r$
- D $(r+s)+t = r+(s+t)$
- E None of these

The inverse operation of addition is

- F Multiplication
- G Subtraction
- H Division
- J Adding the problem again in the reverse order
- K None of these

Which one of the numerals should go in the box to make the sentence true?

$8 \times 7 = 7 \times \square$

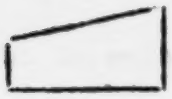
- F 7
- G 8
- H 9
- J 54
- K 56

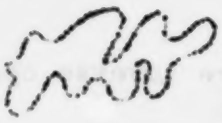
Topic: Geometry

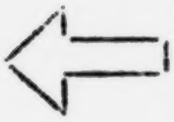
Subtopic: Definitions of Terms

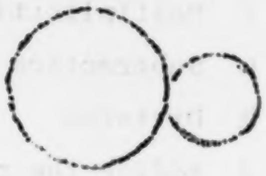
Instructional Objectives	Grade Level 4	5	6
1. To define terms referring to parallel lines and angles	X	X	X
2. To define terms referring to circles	NA	X	X
3. To define terms referring to sets of points	NA	X	X

Which figure is not a simple closed curve?


F


G


H

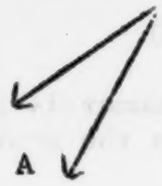

J

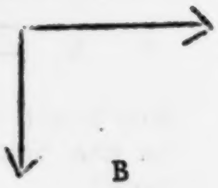
F F J J
G G K None of these
H H

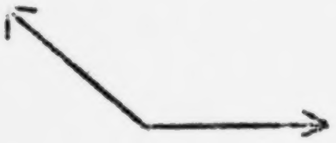
How many endpoints does a line have?

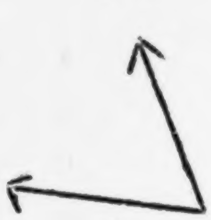
A Infinitely many
B Three endpoints
C Two endpoints
D One endpoint
E No endpoints

Which angle is a right angle?


A


B


C


D

A A D D
B B E None of these
C C

The surface of a table may be thought of as

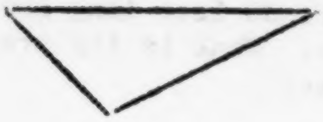
F a ray
G a line
H a square
J a part of a line
K a part of a plane

Topic: Geometry

Subtopic: Recognition of geometric figures

Instructional Objectives	Grade Level 4	Grade Level 5	Grade Level 6
1. To recognize and name two dimensional (plane) figures	X	X	X
2. To recognize and name three dimensional (solid) figures	NA	X	X

Which figure is a triangle?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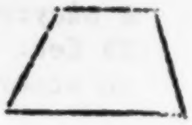
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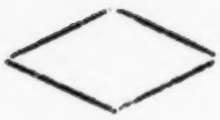
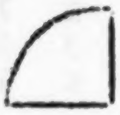
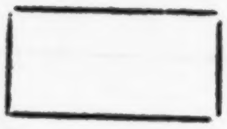
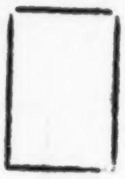

H


J

F F
G G
H H
J J
K None of these

Which figure is a square?


A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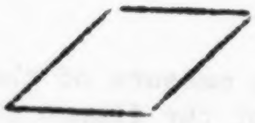
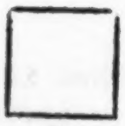
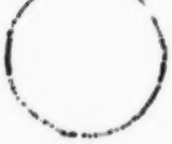
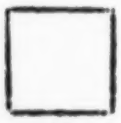

C

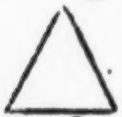

D

A A
B B
C C
D D
E None of these

Which figure is a rectangle?


A


B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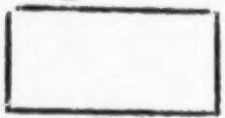
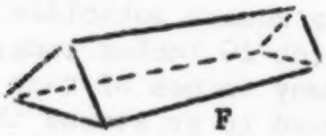

D

A A
B B
C C
D D
E None of these

Which figure is a cube?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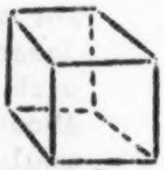
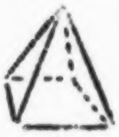
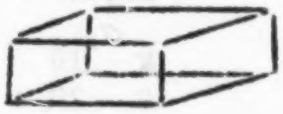

G


H

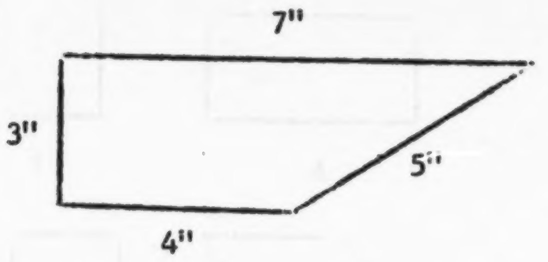

J

F F
G G
H H
J J
K None of these

Topic: Geometry

Subtopic: Measurement of geometric figures

Instructional Objectives	Grade Level 4	 5	 6
1. To find the perimeter of polygons	NA	X	X
2. To find the area of squares, rectangles, triangles, and similar polygons	NA	X	X
3. To find the volume of rectangular solid	NA	NA	X



What is the measure of the perimeter of the figure above?

F 15 inches
G 19 inches
H 20 square inches
J 21 square inches
K 47 inches

For one of her Brownie projects, Janet made a square potholder with each side 10 inches long. About how many inches of tape would she need to go around the edges?

F 10
G 20
H 40
J 100
K None of these

A playroom is 30 feet long and 25 feet wide. What is its area in square feet?

F 55
G 75
H 750
J 1275
K 1750

What is the volume of a rectangular solid which measures 2 feet in length, 4 feet in width, and 3 feet in height?

A 8 cu. ft.
B 12 cu. ft.
C 20 cu. ft.
D 24 cu. ft.
E 36 cu. ft.

Topic: Relations

Subtopic: Equality

Instructional Objectives	Grade Level 4	5	6
1. To demonstrate understanding of equivalence	X	X	X
2. To demonstrate understanding of equality	X	X	X

Sue reported to her scout troop one night that there are four planets larger than the earth and four smaller. If the earth has a diameter of 7900 miles, which one of the following planets is smaller than the earth?

A Venus - diameter 7600 miles
B Uranus - diameter 30,800 miles
C Saturn - diameter 72,400 miles
D Jupiter - diameter 86,700 miles
E None of these

7 + 10 ☐ 10 + 7

Which symbol should be written in the box to make the sentence true?

F <
G >
H ≤
J =
K ~

10 ☐ 9

Which symbol should go in the box to make the sentence true?

A =
B +
C >
D <
E ≤

The newspaper said that Chicago had 1.06 inches of rain on Monday, 1.15 inches on Thursday, and 1.51 inches on Saturday. Which day had the LEAST amount of rain?

F Monday
G Thursday
H Saturday
J Thursday and Saturday
K You can't tell

Topic: Relations

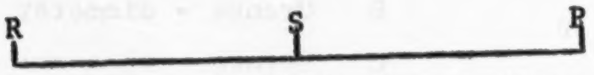
Subtopic: Order relations

Instructional Objectives	Grade Level 4	5	6
1. To demonstrate understanding of order relations using a number line	X	X	X
2. To demonstrate understanding of the order of decimals	X	X	X

A boy scout first walked 5 miles east and then 10 miles west. How many miles and in what direction should he walk to get back to his starting point?

- **A 5 miles east**
- **B 5 miles west**
- **C 15 miles east**
- **D 15 miles west**
- **E He is already at his starting point.**

Robert's birth is marked on the time line below at R. His brother Sam's birth is marked at S, and the present is marked at P. S is midway between R and P.



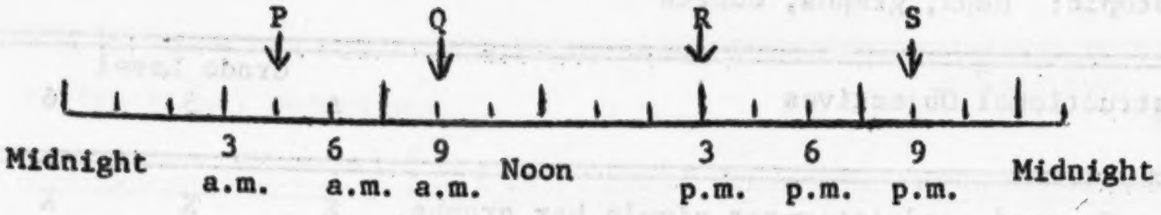
Which one of the following sentence is true?

- **F Robert is younger than Sam.**
- **G Sam and Robert are exactly the same age.**
- **H Sam is now twice as old as Robert.**
- **J Robert is now twice as old as Sam.**
- **K None of these**

Which number has the largest value?

- **A 2.005**
- **B 2.05**
- **C 2.5**
- **D .25**
- **E 25**

Order Relations (cont.)


P
Q
R
S
Midnight
3 a.m.
6 a.m.
9 a.m.
Noon
3 p.m.
6 p.m.
9 p.m.
Midnight

An orderly way to think about events is to arrange them along a time line similar to the one above.

Which one of the following points in Mary's day is NOT shown at the most likely place on the time line above?

A Mary goes to bed at S

B School is out at R

C Mary eats dinner at P

D School starts at Q

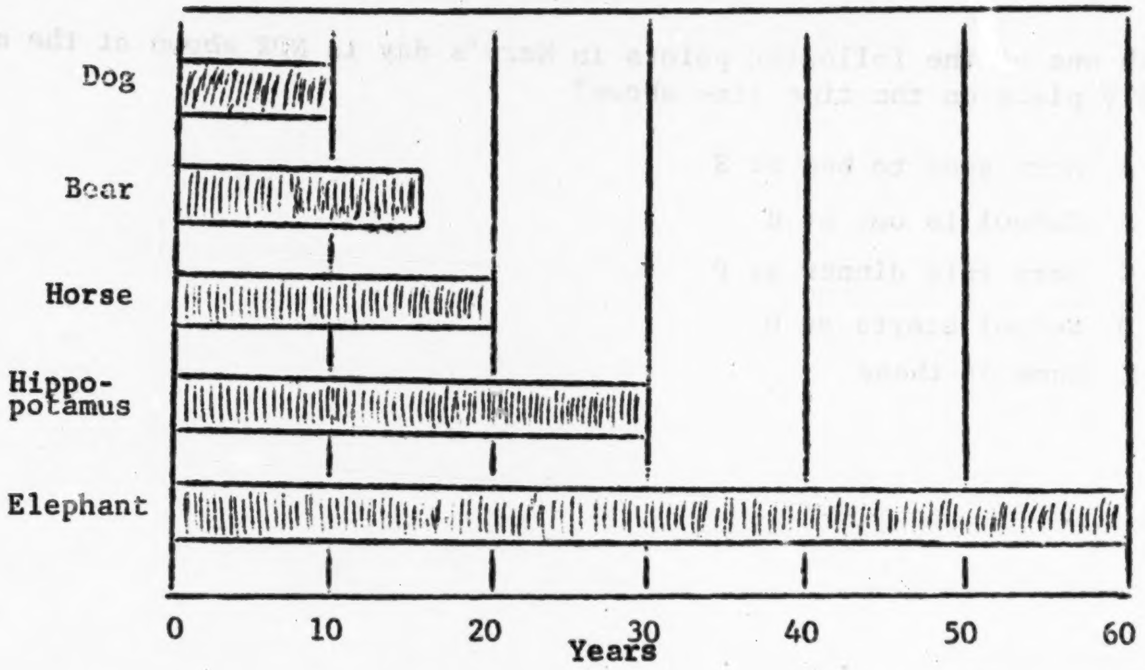
E None of these

Topic: Relations

Subtopic: Maps, graphs, charts

Instructional Objectives	Grade Level 4	5	6
1. To read, and interpret simple bar graphs.	X	X	X

AVERAGE LIFE SPAN OF ANIMALS



Jim said, "No elephant can live to be 70 years old." Bob said, "All elephants live to be at least 50 years old." Tom said, "All elephants live to be 90 years old." According to the graph, who is right?

F Only Jim

G Only Bob

H Only Tom

J Both Jim and Bob

K Neither Jim nor Bob nor Tom

On the average, how many years does a horse live?

A 2

B 10

C 20

D 47

E 57

On the average, a bear lives how many times as long as a dog?

F 1 1/2

G 2

H 5

J 10

K 15

On the average, which animal lives two times as long as a dog?

A Bear

B Horse

C Hippopotamus

D Elephant

E None of these

Topic: Relations

Subtopic: Functions

		Grade Level	
Instructional Objectives	4	5	6
1. To use ratio in solving problems that involve the idea of rate or comparison	X	X	X
2. To solve problems which involve the concept of proportion	X	X	X

Bill's mother planned to use up a big cake at her son's birthday party by cutting equal pieces for his 20 friends. It rained that day and only 10 children were at the party. Bill's mother still planned to use up the whole cake by cutting equal pieces for the 10 children instead. This meant that each serving was

A two times as large as she had planned

B one-fourth as large as she had planned

C one-half as large as she had planned

D four times as large as she had planned

E unequal in size

On a certain map 2 inches stands for one mile. How many inches on the map would stand for a distance of 10 miles?

A 5 C 10 E 20

B 8 D 15

John saves 10 cents a week. Susan saves 15 cents a week. At this rate after how many weeks will Susan have saved 5 times as much as John?

F 1 H 3 K Never

G 2 J 5

Bill and Sarah had a bicycle race from one town to another. Bill took one hour and twenty minutes. Sarah took 80 minutes. Bill's speed was

F $\frac{4}{3}$ of Sarah's speed

G $\frac{3}{2}$ of Sarah's speed

H $\frac{3}{4}$ of Sarah's speed

J $\frac{2}{3}$ of Sarah's speed

K the same as Sarah's speed

Topic: Numeration

Subtopic: Sets

Instructional Objectives	Grade Level 4	5	6
1. To demonstrate an understanding of the concept of sets and set termniology	X	X	X
2. To distinguish among kinds of sets	X	X	X
3. To demonstrate an understanding of union and intersection of sets	X	X	X

A symbol for an empty set is

F $<$

G $\{$

H $\simeq$

J $>$

K $\emptyset$

The union of $\{2, 4, 6,\}$ and $\{3, 6, 9\}$ is

A $\{2, 4, 6, 6\}$

B $\{2, 3, 4, 6, 6, 9\}$

C $\{6\}$

D $\{2, 3, 4, 6, 9\}$

E None of these

A set which is equal to $\{3, 5, 11, 7, 9\}$ is

F $\{35\}$

G $\{2, 4, 10, 6, 8\}$

H $\{11, 9, 7, 5, 3\}$

J $\{3, 5, 7, 9\}$

K None of these

If Set $C = \{6, 7, 8, 9\}$ and Set $D = \{4, 5, 6, 7\}$ find Set $C \cap D$.

A $\{6, 7, 8\}$

B $\{7, 8, 9\}$

C $\{6, 7\}$

D $\{6, 9, 4, 7\}$

E $\{4, 5, 7, 8, 9\}$

Topic: Numeration

Subtopic: Other Numeration Bases

Instructional Objectives	Grade Level 4	5	6
1. To compare base five and base ten	X	X	X
2. To count beyond the square of the base	X	X	X
3. To use other number bases	X	X	X

23_{five} means

A twenty-three

B 2 fives and 3 ones

C 2 tens and 3 ones

D 2 tens and 3 fives

E 23 fives and 0 ones

13 14 15 20 21 22

In what number base is the counting above done?

F $\text{Base}_{\text{three}}$

G $\text{Base}_{\text{five}}$

H Base_{six}

J Base_{ten}

K $\text{Base}_{\text{sixteen}}$

Which is the largest two-place numeral that can be written in base five?

A 44_{five}

B 55_{five}

C 66_{five}

D 99_{five}

E None of these

What numeral in base ten stands for the same number as 440_{five}?

F 44_{ten}

G 84_{ten}

H 88_{ten}

J 120_{ten}

K 220_{ten}

Topic: Numeration

Subtopic: Reading, writing, using whole numbers and decimals

Instructional Objectives	Grade Level 4	 5	 6
1. To read, write, and use whole numbers to nine places	X	X	X
2. To read, write, and use decimals	X	X	X

In the number 5.05, the digit 5 is used twice. Which one of the following is true?

A The "5's" stand for the same amount.

B The "5" at the left stands for an amount 10 times the amount represented by the "5" on the right.

C The "5" at the right stands for an amount larger than the one on the left.

D The "5" at the left stands for an amount 100 times the amount represented by the "5" on the right.

E The "5" at the left stands for an amount 1000 times the amount represented by the "5" on the right.

The numeral 643 has three digits. What does the "6" stand for?

A Six

B Six tens

C The number of hundreds

D The number of six-hundreds

E The number of thousands

The school district paid sixty thousand dollars for new buses. How would you write this amount?

F $60

G $600

H $6,000

J $60,000

K $600,000

Which numeral stands for the smallest number you can write with the digits 4, 9, 1, and 6?

F 1946

G 9164

H 1469

J 1649

K 4961

Topic: Numeration

Subtopic: Roman Numerals

Instructional Objectives	Grade Level 4	5	6
1. To read and write Roman Numerals through LXXX	X	X	X

In the Cathedral Latin School, one of the rooms is numbered XVI. What does XVI mean?

A 10 hundreds + 5 tens + 1

B 10 + 5 + 1

C 10 + 4

D 151

E 61

In Roman Numerals, what letter do we place before X to make 9?

A L

B D

C I

D C

E X

XLV is the same as

F 25

G 45

H 64

J 615

K 1054

In Roman numerals, what letter do we place after X to make 19?

F XI

G IX

H IXX

J LX

K XXI

Topic: Measurement
Subtopic: Concepts

	Grade Level		
Instructional Objectives	4	5	6
1. To demonstrate an understanding of measurement concepts related to the following: size, quantity, distance, time, and weight.	X	X	X

Which unit is a measure of Length?

A A square foot
B An acre
C A square yard
D A mile
E A cubic yard

Which unit of measurement is found in the metric system?

F A pint
G A yard
H A meter
J A ton
K A week

A half-pint is related to a pint the way a

F pint is related to a quart
G pint is related to a gallon
H quart is related to a gallon
J pint is related to a half gallon
K quart is related to five gallons

John is in school every weekday from 8:30 a.m. until 2:45 p.m. How many hours does he spend in school each day?

F 5 1/4
G 5 3/4
H 6
J 6 1/4
K 7 1/4

Topic: Measurement

Subtopic: Conversion of units

Instructional Objectives	Grade Level 4	 5	 6
1. To change measures from one unit to another	X	X	X

John helped clear fallen trees after a storm. He saw a large tree which he guessed was 20 yards long. The tree was actually 65 feet long. John's guess

F was too small

G was exactly right

H was too large

J cannot be compared to the actual length

K was none of these

Betty practices her music lessons 140 minutes each week. How much time will she spend practicing in 4 weeks?

A 3 hours 20 minutes

B 4 hours 20 minutes

C 8 hours 20 minutes

D 9 hours 20 minutes

E 9 hours 80 minutes

Betty has been saving pennies in a jar. She found that she had six hundred thirty-five pennies. This is the same as

A $6.35

B $35.60

C $63.50

D $600.35

E $635.00

David is 42 inches tall. This is the same as

F 3 feet

G 3 feet 6 inches

H 3 feet 9 inches

J 4 feet

K 4 feet 2 inches

Topic: Measurement

Subtopic: Operations without Conversion

Instructional Objectives	Grade Level 4	5	6
1. To solve problems using operations with measures	X	X	X

On a week-end trip George and his father drove 97.3 miles on Friday, 104.8 miles on Saturday, and 124.6 miles on Sunday. How many miles did they drive on the whole trip?

A 215.7

B 226.7

C 326.7

D 2157

E 3667

Amy is 56 inches tall. Joan is 50 inches tall. Amy is how many inches taller than Joan?

A 6

B 16

C 50

D 56

E 106

If erasers cost 5 cents each, how many cents do 7 erasers cost?

F 5

G 7

H 12

J 35

K 45

Janet's mother bought $\frac{1}{2}$ of a quart of peach ice cream and $1\frac{1}{2}$ quarts of chocolate ice cream for Janet's party. How many quarts of ice cream did she buy in all?

A $\frac{3}{4}$

B 1

C 2

D 3

E 4

Topic: Measurement

Subtopic: Operations with Conversion

Instructional Objectives	Grade Level 4	5	6
1. To solve problems which require conversion of units of measure, using all operations	X	X	X

Bill bought 3 pounds of bananas at 8 cents per pound. Which one of the following groups of coins would be the right amount to pay for the bananas?

F 1 dime, 2 nickels, 4 pennies

G 1 quarter, 4 pennies

H 2 dimes, 1 nickel, 4 pennies

J 2 dimes, 2 nickels, 4 pennies

K 3 dimes, 4 pennies

The Riley family uses 4 eggs each day. How many days will 3 dozen eggs last the Riley family?

A 4

B 9

C 12

D 36

E 40

Ellen was 2'8" tall when David was 3'4". At that time, David was how many inches taller than Ellen?

F 4

G 6

H 8

J 14

K 16

Mr. Clark drove his car 1212 miles last year. How many miles on the average did he drive his car each month?

F 11

G 101

H 121.2

J 1200

K 1212

Topic: Fundamental operations with fractions

Subtopic: Concept of fractions

Instructional Objectives	Grade Level 4	 5	 6
1. To find common fractional parts of the whole object and the group	X	X	X
2. To demonstrate understanding of a reciprocal	NA	NA	X
3. To interpret decimals as an extension of one number system	NA	X	X

The reciprocal of $\frac{7}{8}$ is

A 1

B 0

C $\frac{8}{7}$

D $\frac{7}{8}$

E $\frac{49}{64}$

What fraction of the total number of circles is shaded?

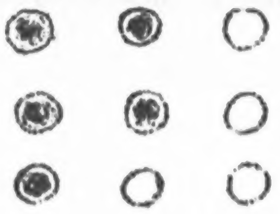
F 5

G $\frac{4}{5}$

H $\frac{4}{9}$

J $\frac{5}{9}$

K 4



Which decimal numeral names 951 hundredths?

A .0951

B .951

C 9.51

D 95.1

E 951.

What part of the rectangle is shaded?

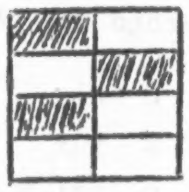
F $\frac{1}{4}$

G $\frac{1}{3}$

H $\frac{3}{8}$

J $\frac{3}{5}$

K $\frac{5}{8}$



Topic: **Fundamental operations with fractions**

Subtopic: **Addition**

Instructional Objectives	Grade Level 4	 5	 6
1. To add like fractions	X	X	X
2. To add decimals	X	X	X
3. To add unlike but related fractions	NA	X	X

Betty and Carol are making dresses for their dolls. Betty bought $\frac{3}{4}$ of a yard of material and Carol bought $\frac{1}{2}$ of a yard of material. How many yards of material did the girls buy altogether?

F $\frac{2}{3}$

G $\frac{5}{6}$

H 1

J $1\frac{1}{4}$

K $1\frac{2}{3}$

$$\begin{array}{r} \frac{1}{3} \\ + \frac{1}{3} \\ \hline \end{array}$$

F $\frac{1}{2}$

G $\frac{2}{3}$

H $\frac{8}{9}$

J $1\frac{1}{3}$

K None of these

$$\begin{array}{r} 8.09 \\ + 24.76 \\ \hline \end{array}$$

F. 22.75

G. 22.85

H. 32.75

J. 32.85

K. None of these

$\frac{1}{6} + \frac{1}{6} =$ (?)

F $\frac{1}{3}$

G $\frac{1}{12}$

H $\frac{2}{12}$

J $\frac{1}{36}$

K None of these

Topic: Fundamental operations with fractions

Subtopic: Subtraction

Instructional Objectives	Grade Level 4	 5	 6
1. To subtract like fractions	X	X	X
2. To subtract decimals	X	X	X
3. To subtract unlike but related fractions	NA	X	X

$$\begin{array}{r} 60.00 \\ -\ 28.74 \\ \hline \end{array}$$

A 21.26

B 31.26

C 42.36

D 48.74

E None of these

$$\begin{array}{r} \frac{5}{7} \\ -\ \frac{2}{7} \\ \hline \end{array}$$

F $\frac{3}{14}$

G $\frac{2}{7}$

H $\frac{3}{7}$

J 3

K None of these

$5 - \frac{2}{5} = (\ ?\ )$

A $\frac{3}{5}$

B 2

C $4\frac{3}{5}$

D $5\frac{3}{5}$

E None of these

Philip had $\frac{3}{4}$ of a gallon of gas. If he used $\frac{1}{2}$ of a gallon in his model plane, how many gallons of gas did he have left?

A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{3}{8}$

D $\frac{1}{2}$

E $1\frac{1}{4}$

Topic: Fundamental operations with fractions

Subtopic: Multiplication

Instructional Objectives	Grade Level 4	 5	 6
1. To multiply a common fraction by a whole number	NA	NA	X
2. To multiply a fraction by a fraction	NA	NA	X
3. To multiply a decimal fraction by a whole number	NA	NA	X
4. To multiply a decimal fraction by a decimal fraction	NA	NA	X

Susan knows the cost of $\frac{1}{3}$ yard of ribbon. To find the cost per yard, she should

- A add $\frac{1}{3}$ three times
- B subtract $\frac{1}{3}$ from the cost per yard
- C multiply the cost of $\frac{1}{3}$ yard by three
- D divide the cost per yard by $\frac{1}{3}$
- E divide the cost of $\frac{1}{3}$ yard by three

$$\begin{array}{r} 1.3 \\ \underline{\times .04} \end{array}$$

- F .0052
- G .052
- H .52
- J 5.20
- K 52.

$$\begin{array}{r} .33 \\ \underline{\times\ 2} \end{array}$$

- A .066
- B .66
- C 6.6
- D 66
- E None of these

$\frac{3}{7} \times \frac{4}{5} = (?)$

- F $\frac{12}{35}$
- G $\frac{15}{28}$
- H $\frac{7}{12}$
- J $\frac{1}{5}$
- K None of these

Topic: Fundamental operations with fractions

Subtopic: Division

Instructional Objectives	Grade Level 4	5	6
1. To divide a whole number by a decimal fraction	NA	NA	X
2. To divide a fraction by a fraction	NA	NA	X

Four pounds and 4 ounces is the same as

- A 4.15 pounds
- B $4\frac{1}{4}$ pounds
- C $4\frac{1}{2}$ pounds
- D $4\frac{3}{4}$ pounds
- E 5 Pounds

Find the correct answer.

$\frac{2}{5} \div \frac{7}{10} =$

- A $\frac{4}{7}$
- B $\frac{2}{7}$
- C $\frac{2}{25}$
- D $\frac{7}{25}$
- E $\frac{7}{50}$

Find the correct answer.

$.4\overline{)\,122}$

- A .035
- B .305
- C 3.05
- D 30.5
- E 305

Which sentence is another way to express the relationship

$$\frac{\frac{3}{4}}{\frac{5}{8}} = n?$$

- F $\frac{3}{4} \div \frac{8}{5} = n$
- G $\frac{3}{4} \div \frac{5}{8} = n$
- H $\frac{4}{3} \times \frac{5}{8} = n$
- J $\frac{4}{3} \times \frac{8}{5} = n$
- K $\frac{3}{4} - \frac{5}{8} = n$

APPENDIX C

SUPPLEMENTARY MATHEMATICS TEST

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SUPPLEMENTARY MATHEMATICS TEST

DIRECTIONS FOR EXAMINER

(Repeat the directions in the SCAT examiners's manual on pages 6 and 7 as you did for SCAT Parts II and IV. When you finish the General Directions Section, say:)

On the front of your booklet, you will find Directions for Part I of the test. Read the directions silently while I read them aloud.

There are 30 problems in Part I of the test. Following each problem there are five suggested answers. Work each problem in your head or on a piece of scratch paper. Then look at the five suggested answers and decide which one is correct. Blacken the space under its letter on the answer sheet.

Sample Problem

$$\begin{array}{r} 4251 \\ +\ \underline{1376} \end{array}$$

A 5527 B 5627 C 2875 D 5628 E None of these

Because the correct answer to the sample problem is 5627, which is lettered B, the space marked B on the answer sheet is blackened. See how it is marked on the answer sheet. Do not make any marks in your test booklet. Do not turn the page until you are told to do so. (Be sure pupils are marking their answers in the correct sections of the answer sheet.)

Are there any questions about what you should do in Part I?

(Pause momentarily to allow questions. Then say:)

There are 30 problems in Part I and you will have 25 minutes to work them. When I say "Begin", turn to the next page and start to work. Ready? Begin.

(Move quietly around the room to make sure that every student is working in the proper part of the test and the answer sheet. At the end of 25 minutes, say:)

Stop! Even if you have not finished Part I you must stop and lay down your pencil.

Turn the page to the directions for Part III.

Read the directions silently while I read them aloud.

There are 30 problems in Part III of the test. Following each problem there are five suggested answers. Work each problem in your head or on a piece of scratch paper. Then look at the five suggested answers and decide which one is correct. Blacken the space under its letter on the answer sheet.

Sample Problem

Jane spent 25 ¢ for candy and 49¢ for a fountain pen. How much did she spend in all?

A 64¢ B 75¢ C 73¢

D 74¢ E None of these

Because the correct answer to the sample problem is 74¢, which is lettered D, the space marked D on the answer sheet is blackened. See how it is marked on the answer sheet. Do not make any marks in your test booklet. Do not turn the page until you are told to do so. (Be sure pupils are marking their answers in the correct section of the answer sheet.)

Are there any questions about what you should do in Part III?

(Pause momentarily to allow questions. Then say:)

There are 30 problems in Part III and you will have 30 minutes to work them. When I say "Begin", turn to the next page and start to work. Ready? Begin!

(Move quietly around the room to make sure that every student is working in the proper part of the test and answer sheet. At the end of 30 minutes, say:)

STOP! Even if you have not finished Part III you must stop and lay down your pencil.

(Collect test books, answer sheets, and IBM pencils.)

MATHEMATICS-PART I

DIRECTIONS

There are 30 problems in Part I of the test. Following each problem there are five suggested answers. Work each problem in your head or on a piece of scratch paper. Then look at the five suggested answers and decide which one is correct. Blacken the space under its letter on the answer sheet.

Sample Problem

$$\begin{array}{r} 4251 \\ +\ \underline{1376} \end{array}$$

A 5527 B 5627 C 2875 D 5628 E None of these

Because the correct answer to the sample problem is 5627, which is lettered B, the space marked B on the answer sheet is blackened. See how it has been marked on the answer sheet. Do not make any marks in your test booklet.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

PART I / TIME: 25 MINUTES

1 Betty has been saving pennies in a jar. She found that she had six hundred thirty-five pennies. This is the same as

A $6.35
B $35.60
C $63.50
D $600.35
E $635.00

2 Which numeral stands for the smallest number you can write with the digits 4, 9, 1, and 6?

F 1946
G 9164
H 1469
J 1649
K 4961

3 Which number has the largest value?

A 2.005
B 2.05
C 2.5
D .25
E 25

4 The school district paid sixty thousand dollars for new buses. How would you write this amount?

F $60
G $600
H $6,000
J $60,000
K $600,000

5 Which decimal numeral names 951 hundredths?

A .0951
B .951
C 9.51
D 95.1
E 951.

6 7 + 10 ☐ 10 + 7

Which symbol should be written in the box to make the sentence true?

F <
G >
H ≤
J =
K ≃

7 10 ☐ 9

Which symbol should go in the box to make the sentence true?

A =
B +
C >
D <
E ≤

8 8 x 7 = 7 x ☐

Which one of the numerals should go in the box to make the sentence true?

F 7
G 8
H 9
J 54
K 56

9 6 x ☐ = 0

Which one of the numerals goes in the box to make the sentence true?

A 0
B 1
C 6
D 10
E $\frac{1}{6}$

GO ON TO THE NEXT PAGE

10 In Roman numerals, what letters are placed after X to make 19?

F XI
G IX
H IXX
J LX
K XXI

11 In Roman numerals, what letter is placed before X to make 9?

A L
B D
C I
D C
E X

12 XLV is the same as

F 25
G 45
H 64
J 615
K 1054

13 In the Cathedral Latin School, one of the rooms is numbered XVI. What does XVI mean?

A 10 hundreds + 5 tens + 1
B 10 + 5 + 1
C 10 + 4
D 151
E 61

14 Find the sum:

6
1
9
5

F 11
G 12
H 21
J 22
K 31

15 The numeral 643 has three digits. What does the "6" stand for?

A Six
B Six tens
C The number of hundreds
D The number of six-hundreds
E The number of thousands

16 The inverse operation of addition is

F Multiplication
G Subtraction
H Division
J Adding the problem again in reverse order
K None of these

17 In the number 5.05, the digit 5 is used twice. Which one of the following is true?

A The "5's" stand for the same amount.
B The "5" at the left stands for an amount 10 times the amount represented by the "5" on the right.
C The "5" at the right stands for an amount larger than the one on the left.
D The "5" at the left stands for an amount 100 times the amount represented by the "5" on the right.
E The "5" at the left stands for an amount 1000 times the amount represented by the "5" on the right.

GO ON TO THE NEXT PAGE.

18 The newspaper said that Chicago had 1.06 inches of rain on Monday, 1.15 inches on Thursday, and 1.51 inches on Saturday. Which day had the LEAST amount of rain?

F Monday
G Thursday
H Saturday
J Thursday and Saturday
K You can't tell

19 The correct answer to the division problem below is

$7\ /\ \overline{630}$

A 7
B 9
C 19
D 70
E None of these

20 Which unit of measurement is found in the metric system?

F A pint
G A yard
H A meter
J A ton
K A week

21 Which sentence expresses the property of associativity in addition?
A $rs = s+r$
B $r(s+t) = (r+s)t$
C $r + s = s + r$
D $(r+s)+t = r+(s+t)$
E None of these

22 A symbol for an empty set is

F $<$
G $\{$
H $\simeq$
J $>$
K $\emptyset$

23 The union of {2,4,6} and {3,6,9} is

A {2,4,6,6}
B {2,3,4,6,6,9}
C {6}
D {2,3,4,6,9}
E None of these

24 A set which is equal to {3,5,11,7,9} is

F {35}
G {2,4,10,6,8}
H {11,9,7,5,3}
J {3,5,7,9}
K None of these

25 If set C = {6,7,8,9} and Set D = {4,5,6,7} find Set $C \cap D$.

A {6,7,8}
B {7,8,9}
C {6,7}
D {6,9,4,7}
E {4,5,6,7,8,9}

26 David is 42 inches tall. This is the same as

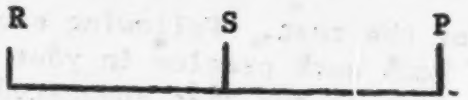
F 3 feet
G 3 feet 6 inches
H 3 feet 9 inches
J 4 feet
K 4 feet 2 inches

27 Which unit is a measure of length?

A A square foot
B An acre
C A square yard
D A mile
E A cubic yard

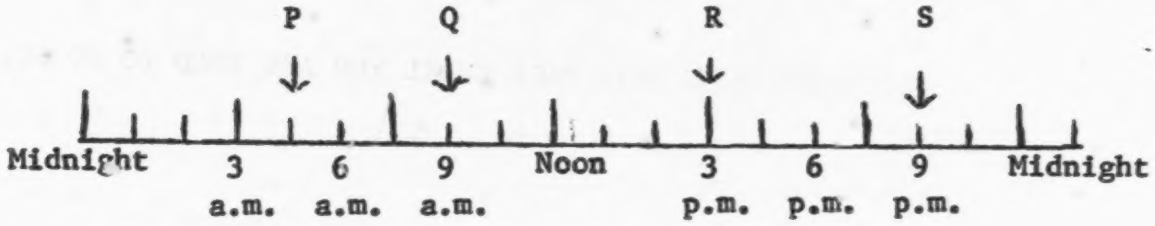
GO ON TO THE NEXT PAGE.

28 Robert's birth is marked on the time line below at R. His brother Sam's birth is marked at S, and the present is marked at P. S is midway between R and P.



Which one of the following sentences is true?

F Robert is younger than Sam.
G Sam and Robert are exactly the same age.
H Sam is now twice as old as Robert.
J Robert is now twice as old as Sam.
K None of these

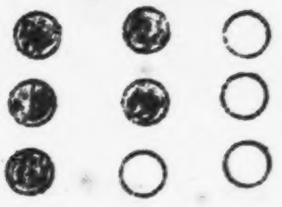


29 An orderly way to think about events is to arrange them along a time line similar to the one above.

Which one of the following points in Mary's day is NOT shown at the most likely place on the time line above?

A Mary goes to bed at S
B School is out at R
C Mary eats dinner at P
D School starts at Q
E None of these

30 What fraction of the total number of circles is shaded?



F 5
G $\frac{4}{5}$
H $\frac{4}{9}$
J $\frac{5}{9}$
K 4

STOP. If you finish before time is called, check your work on this part. Do not go on to next part until you are told to do so.

MATHEMATICS-PART III

DIRECTIONS

There are 30 problems in Part III of the test. Following each problem there are five suggested answers. Work each problem in your head or on a piece of scratch paper. Then look at the five suggested answers and decide which one is correct. Blacken the space under its letter on the answer sheet.

Sample Problem

Jane spent 25¢ for candy and 49¢ for a fountain pen. How much did she spend in all?

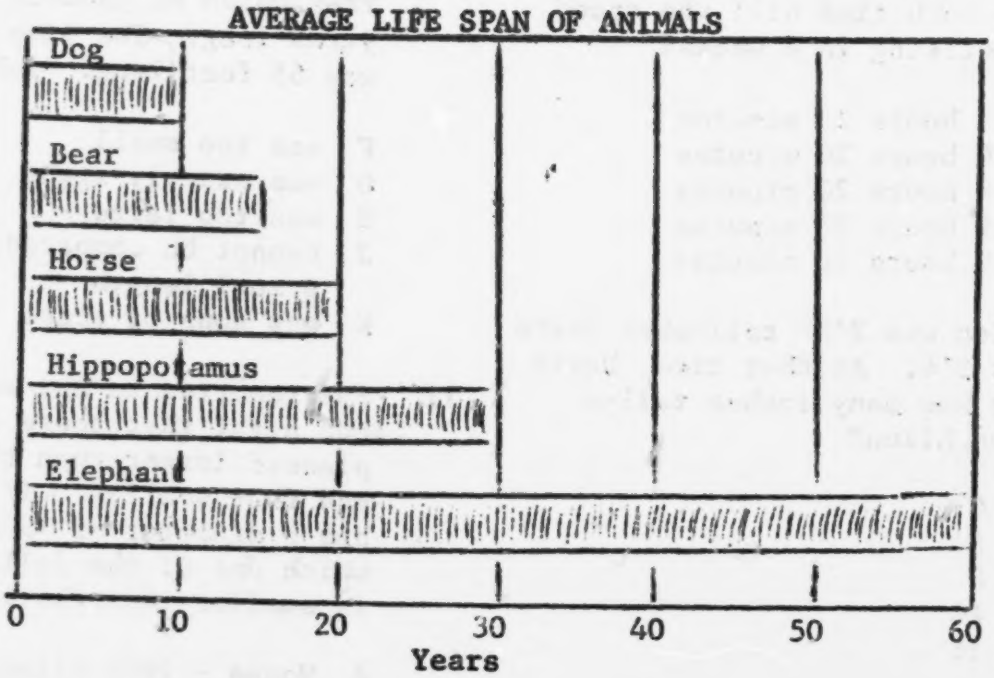
A 64¢ B 75¢ C 73¢
D 74¢ E None of these

Because the correct answer to the sample problem is 74¢, which is lettered D, the space marked D on the answer sheet is blackened. See how it has been marked on the answer sheet. Do not make any marks in your test booklet.

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

PART III /TIME: 30 MINUTES

AVERAGE LIFE SPAN OF ANIMALS



1 On the average, how many years does a horse live?

- A 2
- B 10
- C 20
- D 47
- E 57

2 On the average, a bear lives how many times as long as a dog?

- F 1½
- G 2
- H 5
- J 10
- K 15

3 On the average, which animal lives two times as long as a dog?

- A Bear
- B Horse
- C Hippopotamus
- D Elephant
- E None of these

4 Jim said, "No elephant can live to be 70 years old." Bob said, "All elephants live to be at least 50 years old." Tom said, "All elephants live to be 90 years old." According to the graph, who is right?

- F Only Jim
- G Only Bob
- H Only Tom
- J Both Jim and Bob
- K Neither Jim nor Bob nor Tom

GO ON TO THE NEXT PAGE

5 Betty practices her music lessons 140 minutes each week. How much time will she spend practicing in 4 weeks?

A 3 hours 20 minutes
B 4 hours 20 minutes
C 8 hours 20 minutes
D 9 hours 20 minutes
E 9 hours 80 minutes

6 Ellen was 2'8" tall when David was 3'4". At that time, David was how many inches taller than Ellen?

F 4
G 6
H 8
J 14
K 16

7 Four pounds and 4 ounces is the same as

A 4.15 pounds
B $4\frac{1}{4}$ pounds
C $4\frac{1}{2}$ pounds
D $4\frac{3}{4}$ pounds
E 5 pounds

8 The surface of a table may be thought of as

F a ray
G a line
H a square
J a part of a line
K a part of a plane

9 How many endpoints does a line have?

A Infinitely many
B Three endpoints
C Two endpoints
D One endpoint
E No endpoints

10 John helped clear fallen trees after a storm. He saw a large tree which he guessed was 20 yards long. The tree actually was 65 feet long. John's guess

F was too small
G was exactly right
H was too large
J cannot be compared to the actual length
K was none of these

11 Sue reported to her scout troop one night that there are four planets larger than the earth and four smaller. If the earth has a diameter of 7900 miles, which one of the following planets is smaller than the earth?

A Venus - 7600 miles
B Uranus - 30,800 miles
C Saturn - 72,400 miles
D Jupiter - 86,700 miles
E None of these

12 For one of her Brownie projects, Janet made a square potholder with each side 10 inches long. About how many inches of tape would she need to go around the edges?

F 10
G 20
H 40
J 100
K None of these

13 Which is the largest two-place numeral that can be written in base five?

A 44_{five}
B 55_{five}
C 66_{five}
D 99_{five}
E None of these

GO ON TO NEXT PAGE.

14 What numeral in base ten stands for the same number as 440_{five}?

F 44_{ten}
G 84_{ten}
H 88_{ten}
J 120_{ten}
K 220_{ten}

15 23_{five} means which of the following?

A twenty-three
B 2 fives and 3 ones
C 2 tens and 3 ones
D 2 tens and 3 fives
E 23 fives and 0 ones

16 A playroom is 30 feet long and 25 feet wide. What is its area in square feet?

F 55
G 75
H 750
J 1275
K 1750

17 Find the correct answer.

$$.4\overline{)\,122}$$

A .035
B .305
C 3.05
D 30.5
E 305.

18 Find the correct answer.

$$\begin{array}{r} 1.3 \\ \underline{\times\quad .04} \end{array}$$

F .0052
G .052
H .52
J 5.20
K 52.

19 Find the correct answer.

$$\frac{2}{5} \div \frac{7}{10} =$$

A $\frac{4}{7}$
B $\frac{2}{7}$
C $\frac{2}{25}$
D $\frac{7}{25}$
E $\frac{7}{50}$

20 Which sentence is another way to express the relationship

$$\frac{\frac{3}{4}}{\frac{5}{8}} = n?$$

F $\frac{3}{4} \div \frac{8}{5} = n$
G $\frac{3}{4} \div \frac{5}{8} = n$
H $\frac{4}{3} \times \frac{5}{8} = n$
J $\frac{4}{3} \times \frac{8}{5} = n$
K $\frac{3}{4} - \frac{5}{8} = n$

21 The reciprocal of $\frac{7}{8}$ is

A 1
B 0
C $\frac{8}{7}$
D $\frac{7}{8}$
E $\frac{49}{64}$

GO ON TO NEXT PAGE.

22 13 14 15 20 21 22

In what number base is the counting above done?

F Base$_{\text{three}}$

G Base$_{\text{five}}$

H Base$_{\text{six}}$

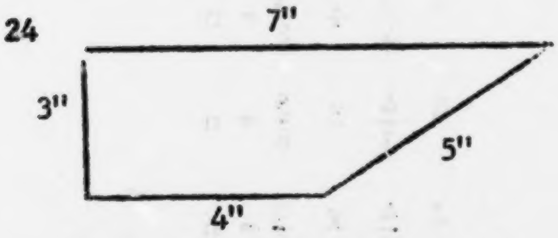
J Base$_{\text{ten}}$

K Base$_{\text{sixteen}}$

23 What is the volume of a rectangular solid which measures 2 feet in length 4 feet in width, and 3 feet in height?

A 8 cubic feet
B 12 cubic feet
C 20 cubic feet
D 24 cubic feet
E 36 cubic feet

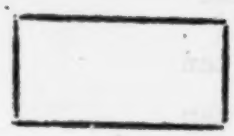
24

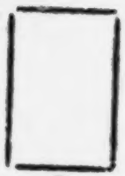


What is the measure of the perimeter of the figure above?

F 15 inches
G 19 inches
H 20 square inches
J 21 square inches
K 47 inches

25 Which figure is a square?


A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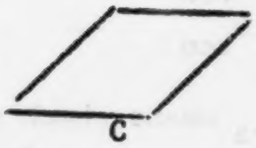
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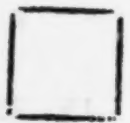
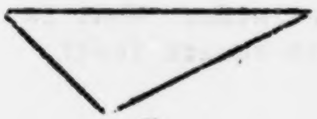
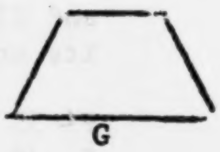

D

A A
B B
C C
D D
E None of these

26 Which figure is a triangle?


F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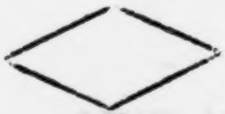
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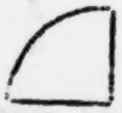

J

F F

G G

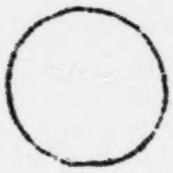
H H

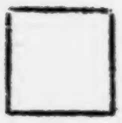
J J

K None of these

GO ON TO THE NEXT PAGE.

27 Which figure is a rectangle?


A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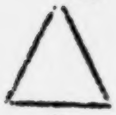
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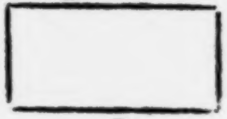
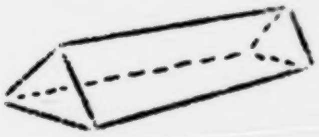

D

A A
B B
C C
D D
E None of these

28 Which figure is a cube?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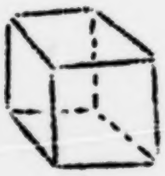
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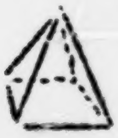
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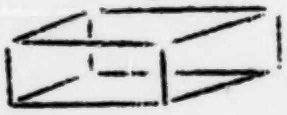
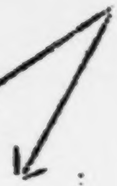
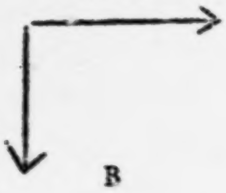

J

F F
G G
H H
J J
K None of these

29 Which angle is a right angle?


A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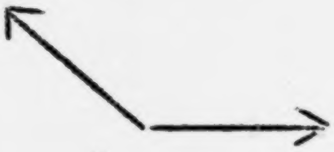
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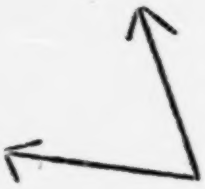
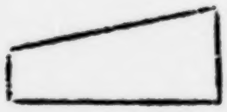
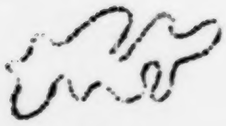

D

A A
B B
C C
D D
E None of these

30 Which figure is NOT a simple closed curve?


F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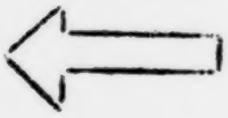
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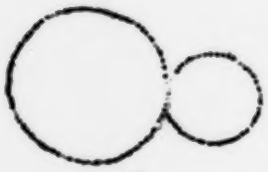
J

F F
G G
H H
J J
K None of these

STOP. If you finish before time is called check your work on this part.

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APPENDIX D

SOURCE AND IDENTIFICATION OF MATHEMATICS TEST ITEMS

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Introduction

Listed below are the source and identification of the items included in the test. They are organized by topic, subtopic, item identification, and location in the supplementary test. The names of the tests have been abbreviated. The abbreviations used are the following.

SCAT - School and College Ability Tests, Form A

STEP - A - Sequential Tests of Educational Progress, Level 4, Form A

STEP - B - Sequential Tests of Educational Progress, Level 4, Form B

CL - Contemporary Mathematics Tests, Lower Elementary Level, Form A

CU - Contemporary Mathematics Tests, Upper Elementary Level, Form A

Topic I
Number, Operations, Assumptions

Subtopics	Item Identification	Part and Item Number: Supplementary Test
A. Addition	SCAT, Pt. II, Item 2 SCAT, Pt. II, Item 3 SCAT, Pt. IV, Item 4 SCAT, Pt. II, Item 5	
B. Subtraction	SCAT, Pt. II, Item 1 SCAT, Pt. II, Item 7 SCAT, Pt. II, Item 11 SCAT, Pt. IV, Item 6	
C. Multiplication	SCAT, Pt. II, Item 14 SCAT, Pt. II, Item 18 SCAT, Pt. II, Item 21 SCAT, Pt. IV, Item 5	
D. Division	SCAT, Pt. II, Item 4 SCAT, Pt. II, Item 19 SCAT, Pt. II, Item 20 SCAT, Pt. II, Item 23	
E. Laws and Generalizations	CL, Item 48 CL, Item 50 CL, Item 48 CL, Item 53	Part I, Item 9 Part I, Item 8 Part I, Item 21 Part I, Item 16

Topic II
Geometry

Subtopics	Item Identification	Part and Item Number: Supplementary Test
A. Definitions of Terms	CL, Item 3	Part III, Item 9
	CL, Item 4	Part III, Item 8
	CL, Item 26	Part III, Item 30
	Author	Part III, Item 29
B. Recognition of Geometric Figures	STEP-A, Pt. I, Item 17	Part III, Item 26
	STEP-A, Pt. I, Item 18	Part III, Item 25
	STEP-B, Pt. I, Item 9	Part III, Item 27
	Author	Part III, Item 28
C. Measurement of Geometric Figures	STEP-A, Pt. I, Item 1	Part III, Item 12
	CU, Item 9	Part III, Item 24
	Author	Part III, Item 23
	Author	Part III, Item 16

Topic III
Relations

A. Equality	STEP-A, Pt. II, Item 12	Part III, Item 11
	STEP-B, Pt. II, Item 3	Part I, Item 18
	CU, Item 14	Part I, Item 7
	CU, Item 42	Part I, Item 6
B. Order Relationships	SCAT, Pt. IV, Item 25	
	STEP-B, Pt. I, Item 21	Part I, Item 29
	STEP-B, Pt. I, Item 25	Part I, Item 28
	Author	Part I, Item 3
C. Map, Graphs, Charts	STEP-B, Pt. II, Item 10	Part III, Item 1
	STEP-B, Pt. II, Item 11	Part III, Item 2
	STEP-B, Pt. II, Item 12	Part III, Item 3
	STEP-B, Pt. II, Item 13	Part III, Item 4
D. Functions	SCAT, Pt. IV, Item 15	
	SCAT, Pt. IV, Item 16	
	SCAT, Pt. IV, Item 21	
	SCAT, Pt. IV, Item 24	

Topic IV
Numeration

A. Sets	Author	Part I, Item 22
	Author	Part I, Item 23
	Author	Part I, Item 24
	Author	Part I, Item 25

Numeration (cont.)

Subtopics	Item Identification	Part and Item Number: Supplementary Test
B. Other Number Bases	CL, Item 36	Part III, Item 15
	CL, Item 38	Part III, Item 22
	CU, Item 29	Part III, Item 14
	CU, Item 37	Part III, Item 13
C. Reading, Writing, Using Numbers	STEP-A, Pt. I, Item 10	Part I, Item 4
	STEP-B, Pt. II, Item 14	Part I, Item 15
	STEP-B, Pt. II, Item 15	Part I, Item 17
	CL, Item 47	Part I, Item 2
D. Roman Numerals	STEP-A, Pt. I, Item 16	Part I, Item 13
	Author	Part I, Item 10
	Author	Part I, Item 11
	Author	Part I, Item 12

Topic V
Measurement

Subtopics	Item Identification	Part and Item Number: Supplementary Test
A. Concepts	SCAT, Pt. IV, Item 12	
	SCAT, Pt. IV, Item 22	
	CL, Item 7	Part I, Item 27
	Author	Part I, Item 20
B. Conversion of Units	STEP-A, Pt. I, Item 5	Part I, Item 26
	STEP-B, Pt. I, Item 17	Part I, Item 1
	STEP-B, Pt. II, Item 9	Part III, Item 10
	Author	Part III, Item 5
C. Operations-No Conversion	SCAT, Pt. IV, Item 1	
	SCAT, Pt. IV, Item 2	
	SCAT, Pt. IV, Item 11	
	SCAT, Pt. IV, Item 13	
D. Operations-With Conversion	SCAT, Pt. IV, Item 8	
	SCAT, Pt. IV, Item 18	
	SCAT, Pt. IV, Item 19	
	STEP-A, Pt. I, Item 6	

Topic VI
Fundamental Operations with Fractions

Subtopics	Item Identification	Part and Item Number: Supplementary Test
A. Concepts	SCAT, Pt. IV, Item 14	
	CL, Item 32	Part I, Item 30
	CU, Item 19	Part I, Item 5
	CU, Item 31	Part III, Item 21

Fundamental Operations with Fractions (cont.)

Subtopics	Item Identification	Part and Item Number: Supplementary Test
B. Addition	SCAT, Pt. II, Item 6	
	SCAT, Pt. II, Item 10	
	SCAT, Pt. II, Item 22	
	SCAT, Pt. IV, Item 20	
C. Subtraction	SCAT, Pt. II, Item 12	
	SCAT, Pt. II, Item 17	
	SCAT, Pt. II, Item 25	
	SCAT, Pt. IV, Item 17	
D. Multiplication	SCAT, Pt. II, Item 15	
	SCAT, Pt. II, Item 24	
	SCAT, Pt. IV, Item 23	
	Author	Part III, Item 18
E. Division	Author	Part III, Item 7
	CU, Item 20	Part III, Item 20
	Author	Part III, Item 17
	Author	Part III, Item 19

APPENDIX E

DESCRIPTIVE AND INFERENTIAL STATISTICS: MEANS AND VARIANCES FOR THE SUBJECTS' PERFORMANCE ON THE MATHEMATICS TEST

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Means and Standard Deviations: Fourth, Fifth, and Sixth Grade Groups' Performance on the Mathematics Test

		Grade 4 n = 244		Grade 5 n = 261		Grade 6 n = 240	
	Variable	$\bar{X}$	s	$\bar{X}$	s	$\bar{X}$	s
Number, Operations, Assumptions	Y1 Addition	3.16	.96	3.24	.88	3.27	.89
	Y2 Subtraction	3.11	1.00	3.25	.92	3.27	.83
	Y3 Multiplication	1.87	1.22	2.59	1.19	3.19	1.04
	Y4 Division	1.41	.94	2.02	1.13	2.70	1.12
	Y5 Laws and Generalizations	2.94	.96	2.67	.96	2.91	.80
	Y6 Total Topic	12.50	3.34	13.75	3.39	15.54	3.04
Geometry	Y7 Definitions of Terms	--	--	--	--	--	--
	Y8 Recognition of Geometric Figures	2.99	1.04	2.94	1.05	3.26	.96
	Y9 Measurement of Geometric Figures	--	--	1.95	1.14	2.80	1.17
	Y10 Total Topic	4.73	1.56	6.79	2.36	7.96	2.39
Relations	Y11 Equality	3.17	.96	3.02	1.06	3.23	.91
	Y12 Order Relations	1.12	1.02	1.68	1.14	2.27	1.27
	Y13 Maps, Graphs, Charts	2.45	1.06	2.46	1.02	2.78	.96

Means and Standard Deviations (Continued)

		Grade 4 n = 244		Grade 5 n = 261		Grade 6 n = 240	
	Variable	$\overline{X}$	S	$\overline{X}$	S	$\overline{X}$	S
Relations (Continued)	Y14 Functions	--	--	2.28	1.17	2.67	1.18
	Y15 Total Topic	8.68	2.75	9.43	3.06	10.94	3.12
Numeration	Y16 Sets	2.64	1.07	2.13	1.09	2.21	1.03
	Y17 Number Bases	1.46	1.29	1.89	1.37	1.52	1.33
	Y18 Reading, Writing, Using Numbers	2.26	1.09	2.49	1.10	2.85	1.12
	Y19 Roman Numerals	1.98	1.18	2.31	1.30	2.62	1.21
	Y20 Total Topic	8.33	3.32	8.83	3.48	9.19	3.15
Measurement	Y21 Concepts of Measurement	1.70	1.17	2.03	1.20	2.40	1.20
	Y22 Conversion of Units	2.04	1.03	2.22	1.10	2.73	1.10
	Y23 Operations -- No Conversion	3.00	1.04	3.38	1.00	3.61	.77
	Y24 Operations -- With Conversion	1.62	1.10	1.89	1.17	2.35	1.10
	Y25 Total Topic	8.37	3.15	9.52	3.37	11.09	3.31

Means and Standard Deviations (Continued)

	Variable	Grade 4 n = 244		Grade 5 n = 261		Grade 6 n = 240	
		$\bar{X}$	S	$\bar{X}$	S	$\bar{X}$	S
Fundamental Operations with Fractions	Y26 Concepts	1.55	1.05	1.93	1.04	3.13	.99
	Y27 Addition	--	--	2.60	1.03	3.16	.97
	Y28 Subtraction	1.58	1.91	2.39	1.12	3.06	1.04
	Y29 Multiplication	--	--	--	--	2.78	1.11
	Y30 Division	--	--	--	--	2.06	1.08
	Y31 Total Topic	4.95	2.22	6.93	2.53	14.14	3.90

Results of Analyses of Variance Among Means: Fourth, Fifth, and Sixth Grade Groups' Performance on the Mathematics Test

Variable	Source of Variation	Degrees of Freedom	Mean Square
Y1 Addition	Between	2	.68
	Within	742	.83
Y2 Subtraction	Between	2	7.82***
	Within	742	.85
Y3 Multiplication	Between	2	105.54***
	Within	742	1.33
Y4 Division	Between	2	101.39***
	Within	742	1.15
Y5 Laws and Generalizations	Between	2	5.76**
	Within	742	.85
Y6 Total Topic: Number, Operations, Assumptions	Between	2	563.52***
	Within	742	10.64
Y7 Definitions of Geometric Terms	Between	--	--
	Within	--	--
Y8 Recognition of Geometric Figures	Between	2	7.16**
	Within	742	1.03
Y9 Measurement of Geometric Figures	Between	2	102.46***
	Within	742	1.18
Y10 Total Topic: Geometry	Between	--	--
	Within	--	--
Y11 Equality	Between	2	3.10*
	Within	742	.94
Y12 Order Relations	Between	2	79.73***
	Within	742	1.32
Y13 Maps, Graphs, Charts	Between	2	8.31***
	Within	742	1.02
Y14 Functions	Between	--	--
	Within	--	--
Y15 Total Topic: Relations	Between	2	320.13***
	Within	742	8.90

Results of Analyses of Variance (Continued)

Variable	Source of Variation	Degrees of Freedom	Mean Square
Y16 Sets	Between Within	2 742	18.32*** 1.14
Y17 Number Bases	Between Within	2 742	14.12*** 1.77
Y18 Reading, Writing, Using Numbers	Between Within	2 742	21.20*** 1.21
Y19 Roman Numerals	Between Within	2 742	24.59*** 1.52
Y20 Total Topic: Numeration	Between Within	2 742	45.17*** 11.06
Y21 Concepts of Measurement	Between Within	2 742	29.25*** 1.42
Y22 Conversion of Units	Between Within	2 742	31.62*** 1.16
Y23 Operations - No Conversion	Between Within	2 742	22.57*** .90
Y24 Operations - With Conversion	Between Within	2 742	32.68*** 1.27
Y25 Total Topic: Measurement	Between Within	2 742	451.69*** 10.76
Y26 Concepts -- Fractions	Between Within	2 742	164.14*** 1.05
Y27 Addition -- Fractions	Between Within	-- --	-- --
Y28 Subtraction -- Fractions	Between Within	2 742	124.71*** 1.13
Y29 Multiplication -- Fractions	Between Within	-- --	-- --
Y30 Division -- Fractions	Between Within	-- --	-- --
Y31 Total Topic: Fractions	Between Within	-- --	-- --

*Associated P < .05; **Associated P < .01; ***Associated P < .001.

Results of Paired Comparisons Between Means and Variances: Fourth, Fifth, and Sixth Grade Groups' Performance on the Mathematics Test[a,b]

Variable	Comparison	$F(S^2/_S2)$	$S_{\overline{X}-\overline{X}}$	$\overline{X} - \overline{X}$	Relationship
Y1 Addition	G4:G5	1.18	--	--	G5=G4
	G4:G6	1.15	--	--	G6=G4
	G5:G6	1.02	--	--	G6=G5
Y2 Subtraction	G4:G5	1.20	.06	.13*	G5>G4
	G4:G6	1.47	.06	.36***	G6>G4
	G5:G6	1.23	.06	.22***	G6>G5
Y3 Multiplication	G4:G5	1.06	.08	.72***	G5>G4
	G4:G6	1.38	.07	1.32***	G6>G4
	G5:G6	1.31	.07	.60***	G6>G5
Y4 Division	G4:G5	1.44	.06	.62***	G5>G4
	G4:G6	1.42	.07	1.29***	G6>G4
	G5:G6	1.02	.07	.68***	G6>G5
Y5 Laws & Generalizations	G4:G5	1.00	.06	.28***	G5<G4
	G4:G6	1.44	.06	.03	G6=G4
	G5:G6	1.45	.06	.24***	G6>G5
Y6 Total Topic : Number, Operations, Assumptions	G4:G5	1.03	.21	1.25***	G5>G4
	G4:G6	1.21	.21	3.04***	G6>G4
	G5:G6	1.25	.20	1.78***	G6>G5

Results of Paired Comparisons (Continued)

Variable	Comparison	$F(S^2/s^2)$	$S_{\overline{X}-\overline{X}}$	$\overline{X}-\overline{X}$	Relationship
Y7 Definitions of Geometric Terms	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	--	--	--	--
Y8 Recognition of Geometric Figures	G4:G5	1.03	.07	.05	G5=G4
	G4:G6	1.17	.06	.27***	G6>G4
	G5:G6	1.20	.06	.32***	G6>G5
Y9 Measurement of Geometric Figures	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	1.07	.07	.85***	G6>G5
Y10 Total Topic: Geometry	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	--	--	--	--
Y11 Equality	G4:G5	1.15	.06	.16*	G5<G4
	G4:G6	1.11	.06	.06	G6=G4
	G5:G6	1.28	.06	.21***	G6>G5
Y12 Order Relations	G4:G5	1.26	.07	.56***	G5>G4
	G4:G6	1.56	.07	1.15***	G6>G4
	G5:G6	1.24	.08	.59***	G6>G5
Y13 Maps, Graphs, Charts	G4:G5	1.08	.07	.01	G5=G4
	G4:G6	1.21	.07	.32***	G6>G4
	G5:G6	1.12	.06	.32***	G6>G5

Results of Paired Comparisons (Continued)

Variable	Comparison	$F(S^2/S^2)$	$S_{\overline{X}} - \overline{X}$	$\overline{X} - \overline{X}$	Relationship
Y14 Functions	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	1.03	.07	.39***	G6>G5
Y15 Total Topic: Relations	G4:G5	1.24	.18	.74***	G5>G4
	G4:G6	1.28	.19	2.26***	G6>G4
	G5:G6	1.04	.20	1.51***	G6>G5
Y16 Sets	G4:G5	1.05	.07	.50***	G5<G4
	G4:G6	1.06	.07	.43***	G6<G4
	G5:G6	1.12	.07	.08	G6=G5
Y17 Number Bases	G4:G5	1.13	.08	.43***	G5>G4
	G4:G6	1.06	.08	.06	G6=G4
	G5:G6	1.08	.09	.38***	G6<G5
Y18 Reading, Writing, Using Numbers	G4:G5	1.01	.07	.23**	G5>G4
	G4:G6	1.05	.07	.59***	G6>G4
	G5:G6	1.04	.07	.36***	G6>G5
Y19 Roman Numerals	G4:G5	1.20	.08	.33***	G5>G4
	G4:G6	1.05	.08	.63***	G6>G4
	G5:G6	1.15	.08	.30***	G6>G5
Y20 Total Topic: Numeration	G4:G5	1.10	.21	.50*	G5>G4
	G4:G6	1.12	.21	.86***	G6>G4
	G5:G6	1.23	.21	.36	G6=G5

Results of Paired Comparisons (Continued)

Variable	Comparison	$F(S^2/S^2)$	$S_{\bar{X}-\bar{X}}$	$\bar{X}-\bar{X}$	Relationship
Y21 Concepts of Measurement	G4:G5	1.05	.08	$.33^{***}$	G5>G4
	G4:G6	1.06	.08	$.70^{***}$	G6>G4
	G5:G6	1.00	.08	$.37^{***}$	G6>G5
Y22 Conversion of Units	G4:G5	1.14	.07	$.18^{**}$	G5>G4
	G4:G6	1.14	.07	$.70^{***}$	G6>G4
	G5:G6	1.00	.07	$.51^{***}$	G6>G5
Y23 Operations -- No Conversion	G4:G5	1.08	.06	$.38^{***}$	G5>G4
	G4:G6	1.81^{*}	.06	$.60^{***}$	G6>G4
	G5:G6	1.68^{*}	.06	$.23^{***}$	G6>G5
Y24 Operations -- With Conversion	G4:G5	1.13	.07	$.27^{***}$	G5>G4
	G4:G6	1.00	.07	$.72^{***}$	G6>G4
	G5:G6	1.13	.07	$.46^{***}$	G6>G5
Y25 Total Topic: Measurement	G4:G5	1.14	.21	1.16^{***}	G5>G4
	G4:G6	1.10	.21	2.72^{***}	G6>G4
	G5:G6	1.04	.21	1.57^{***}	G6>G5
Y26 Concepts of Fractions	G4:G5	1.02	.07	$.39^{***}$	G5>G4
	G4:G6	1.13	.07	1.58^{***}	G6>G4
	G5:G6	1.11	.06	1.19^{***}	G6>G5
Y27 Addition of Fractions	G4:G5	--	--	--	--
	G4:G6	--	--	--	--
	G5:G6	1.13	.06	$.56^{***}$	G6>G5

Results of Paired Comparisons (Continued)

Variable	Comparison	$F(S^2/S^2)$	$S_{\overline{X}-\overline{X}}$	$\overline{X}-\overline{X}$	Relationship
Y28	G4:G5	1.24	.07	.81***	G5>G4
Subtraction of Fractions	G4:G6	1.07	.07	1.43***	G6>G4
	G5:G6	1.16	.07	.62***	G6>G5
Y29	G4:G5	--	--	--	--
Multiplication of Fractions	G4:G6	--	--	--	--
	G5:G6	--	--	--	--
Y30	G4:G5	--	--	--	--
Division of Fractions	G4:G6	--	--	--	--
	G5:G6	--	--	--	--
Y31	G4:G5	--	--	--	--
Total Topic Fractions	G4:G6	--	--	--	--
	G5:G6	--	--	--	--

*Associated $P < .05$; **Associated $P < .01$; ***Associated $P < .001$.

[a] The degrees of freedom for the F tests of homogeneity of variance (MS within) were the following: Grade 4--243; Grade 5--260; Grade 6--239.

[b] The degrees of freedom for the t tests of differences between means were the following for these comparisons: Grade 4 and Grade 5--503; Grade 4 and Grade 6--482; Grade 5 and Grade 6--499.

APPENDIX F

PRINCIPAL AXES COORDINATES FOR TOPICS

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PRINCIPAL AXES COORDINATES FOR TOPICS

Table 1A

Principal Axes Coordinates for Grade 4 (N = 244)

	Dimension	
Topic	1	2
1	-811	1379
2	1780	337
3	-114	-721
4	-490	-580
5	-365	-414

Table 2A

Principal Axes Coordinates for Grade 5 (N = 261)

	Dimension	
Topic	1	2
1	240	1401
2	1770	-522
3	-672	-247
4	-640	-325
5	-698	-306

Table 3A

Principal Axes Coordinates for Grade 6 (N = 240)

	Dimension	
Topic	1	2
1	-605	1723
2	1345	296
3	45	-686
4	-637	-978
5	-149	-355

Table 4A

Principal Axes Coordinates for Six Topics in Grade 6

	Dimension	
Topic	1	2
1	-274	1795
2	1396	-175
3	275	-1135
4	-1155	-956
5	-158	212
6	-84	260